

The Impact of Thin-Capitalization and Earnings Stripping Rules in the EU-15 on the Tax Shield

Carmen Bachmann*

*University of Leipzig
Institute for Accounting, Finance and Taxation
Grimmaische Strasse 12, 04109 Leipzig, Germany
Tel: +49 341 9733-591, Fax: +49 341 9733-599
E-mail: Carmen.Bachmann@uni-leipzig.de*

Alexander Lahmann

*HHL Graduate School of Management
Junior Professorship for M&A in SME
Jahnallee 59, 04109 Leipzig, Germany
Tel: +49 341 9851-665, Fax: +49 341 9851-689
E-mail: Alexander.Lahmann@hhl.de*

Carolin Schuler

*University of Leipzig
Institute for Accounting, Finance and Taxation
Grimmaische Strasse 12, 04109 Leipzig, Germany
Tel: +49 341 9733-590, Fax: +49 341 9733-599
E-mail: Carolin.Schuler@uni-leipzig.de*

Abstract

Several countries within the EU-15 group limit the tax deductibility of interest payments for intragroup financing by thin-capitalization rules or following a recent trend by earnings stripping rules that limit the deductibility on a broader basis. This paper aims at deriving a tax shield valuation framework considering a possible limitation of the tax deductibility of interest imposed by the tax code. We compare the obtained pricing equations and their impact on the tax shield value. Finally, we show that the inclusion of interest limitation rules imply a remarkable reduction of the tax shield value especially for highly indebted companies.

Keywords: *Tax shield, Firm valuation, Thin-capitalization rules, Earnings stripping rules*

JEL classification: *G12, G31, G32, K34*

*Corresponding author. University of Leipzig, Institute for Accounting, Finance and Taxation, Grimmaische Strasse 12, 04109 Leipzig, Germany, E-mail: Carmen.Bachmann@uni-leipzig.de

1. Introduction

Since the financial crisis in 2008 / 2009 politicians and the general public discuss limiting the tax deductibility of interest payments in order to render less attractive excessive debt financing. In 2010 the President's Economic Advisory Board stated in their report on tax reform options '(...) a limitation on the net interest deductibility would lessen the bias against equity financing (...), thereby reducing the leverage of firms and the likelihood of future financial distress.' In Europe similar statements have been made: 'the crisis is most definitely the result of excessive debt (...)' (see Rasmussen (February 2009)). This kind of topic is not new. Already in the early 1990s several governments introduced or at least discussed the limitation of the tax deductibility of interest expenses by thin-capitalization rules.

Among the first, in 2008, the German government limited the tax deductibility of interest payments by introducing an earnings stripping rule, followed by several other EU countries. In contrast to the thin-capitalization rules, these earnings stripping rules are significantly more comprehensive in terms of their applicability. The thin-capitalization rules usually limit the intragroup tax deductibility of interest payments made by foreign subsidiaries and therefore these rules are restricted to a very small field of application. As a response to a decision by the European Court of Justice (ECJ) in 2002, many EU countries extended their rules to domestic intragroup interest payments. As depicted in Table 1, in the meantime several EU-15 countries dismissed their old thin-capitalization rules for adopting an earnings stripping / interest deductibility rule.¹ These rules control the tax deductibility of interest expenses for almost all types of firms and debt.

Table 1
Timeline of Thin-Capitalization and Earnings Stripping Rules in the EU-15

This table shows for countries within the EU-15 group the timeline whether thin-capitalization or earnings stripping rules are applicable. It starts with the year 2004 as the ECJ decision with respect to the then prevailing thin-capitalization rules became effective on January 1, 2004. For example in the case of Denmark (DEN), the table shows that from 2004 until 2007 a thin-capitalization rule was applicable and since 2008 by an earnings stripping rule.

<i>Earnings stripping / interest deductibility rule</i>	GER, DEN, ITA				ESP	FRA, POR	NED, ¹	FIN, GRE
<i>Thin-capitalization rule for all shareholders</i> BEL, DEN ² , FRA, GER, ITA, NED, UK ³	FRA ⁴				GRE			
<i>Thin-capitalization rule for foreign shareholders</i> POR, ESP ⁴						POR ⁵		
2004	2007	2008	2009	2010	2012	2013	2014	

¹ Interest deductibility rule.

² Supplemented by an 'interest ceiling rule' in 2008.

³ With the Finance Act 2004 (July 22, 2004) abolition of a thin-capitalization rule, since 2010 worldwide debt cap (WWDC).

⁴ Introduction of complex thin-capitalization rule.

⁵ Indicating thin-capitalization rule for non-EU shareholders.

Several empirical studies (see, among others, Weichenrieder and Windschbauer (2008), Overesch and Wamser (2010), or Buettner et al. (2012)) confirm that the tax planning behaviour with respect to intercompany debt is influenced by regulations limiting the tax deductibility of interest payments. Studies regarding the earnings stripping rule (see, for example, Knauer and Sommer (2012)) indicate that these rules reduce tax savings due to debt financing. Therefore it is surprising that there is still a lack of models considering these tax regulations for pricing the net benefits of interest payments on debt. The foundation for the standard pricing techniques goes back to the seminal papers of Modigliani and Miller (1958) and (1963) and was further specified by Myers (1974) who introduced the adjusted present value (APV) method.

¹ In the following the term earnings stripping rule is used.

The main portion of this literature intensively discusses the proper discounting of the tax savings with respect to the standard financing policies proposed by *Modigliani* and *Miller* as well as Miles and Ezzell (1980) and (1985) (see for the extensive discussion, for example, Fernandez (2004), Fieten et al. (2005), Arzac and Glosten (2005), Cooper and Nyborg (2006) and recently Massari et al. (2007) and Dempsey (2013)). Since the tax shield accounts for a major part of the overall firm value (see, e.g., Graham (2000)), it is important to analyze the influencing parameters. Recently some important articles started discussing the decreasing effect of default on the tax shield value (see, for example, Cooper and Nyborg (2008), Molnár and Nyborg (2013) or Couch et al. (2012)). Even though the aforementioned is important and the results obtained are beyond questioning, only few articles started considering the tax regulations that limit by law the tax deductibility of interest payments (see, e.g., Knauer et al. (2014)). Obviously, the tax limitations already have an effect on the tax savings before default occurs. Since a correct pricing of the tax savings is vital for practical valuation settings and has to serve as yardstick for empirical studies, it is important to adjust the standard tax shield pricing methods accordingly.

This paper aims at deriving a tax shield valuation framework that is able to determine the effects of thin-capitalization and the recently introduced earnings stripping rules in the EU-15² countries. Hence we can demonstrate the impact on the enterprise value and on the decision between debt and equity financing. Further we provide an overview on the respective tax regulations in the EU-15 countries. Over and above we allow for personal taxes in our framework, as they have a strong impact on the tax advantage of debt (see, e.g., Miller (1977)).

The paper proceeds as follows. Section 2 describes the APV approach as model framework and a binomial lattice modelling the evolution of the firm's free cash flows. Section 3 and 4 derive the respective tax shield pricing models considering the cases of thin-capitalization and earnings stripping rules. Section 5 presents numerical examples that are used for comparing the impact of the different tax rules on the tax shield value and Section 6 concludes. The derivation of the tax shields and further demonstrations are given in the 'Appendix'.

2. The general Model Setting

2.1. Standard tax shield valuation

Let us consider the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and the time interval $[t, T]$, where $T \rightarrow \infty$ is possible. The time interval can be partitioned in N periods of equal length $\Delta t = \frac{T-t}{N}$, where an arbitrary subsequent period $t+1$ is defined by $t+1 = t + \Delta t$. The market is assumed to be free of arbitrage and the existence of a - to the subjective probability measure \mathbb{P} equivalent - risk-neutral probability measure \mathbb{Q} is presupposed. Throughout our model analysis we consider a levered firm whose operating assets generate in every future period s , with $s > t$, an uncertain (unlevered) free cash flow stream FCF^U . The free cash flows are assumed to evolve according to a simple recombining binomial lattice. Consequently, the unlevered free cash flows increase between two arbitrary periods t and $t+1$ by the factor u with a probability p or decrease by the factor d with probability $1-p$:

$$FCF_{t+1}^U = \begin{cases} u \cdot FCF_t^U & , \text{up-movement,} \\ d \cdot FCF_t^U & , \text{down-movement.} \end{cases} \quad (1)$$

An important feature of this recombining binomial lattice is that moving first up and then down or first down and then up results in the same state dependent value $FCF_t^U \cdot u \cdot d = FCF_t^U \cdot d \cdot u$.

²We concentrate on this specific countries as they had already been EU member states in 2004 and therefore enables comparison between thin-capitalization and earnings stripping rules.

The standard approach for valuing tax shields by using the APV approach (see for example Myers (1974) or Modigliani and Miller (1958)) presumes that a levered firm with value V_t^L performs a financing policy with certain (non-stochastic) debt levels D_s in every arbitrary period s , with $s \geq t$. We will define this policy as autonomous financing. In order to account for tax savings due to the tax deductibility of interest payments on debt the levered firm value conditional on the available information in period t is determined by adding on top of the value of an otherwise identical but unlevered firm the present value of the future period-specific tax savings TS_t according to

$$V_t^L = \sum_{s=t+1}^T \frac{E_t[FCF_s^U]}{(1+r_U)^{s-t}} + \sum_{s=t+1}^T \frac{E_t[TS_s]}{(1+r_D)^{s-t}} \quad (2)$$

or in a more explicit form by solely considering corporate taxes τ_C

$$V_t^L = V_t^U + \sum_{s=t+1}^T \frac{E_t[\tau_C \cdot r_D \cdot D_{s-1}]}{(1+r_D)^{s-t}}, \quad (3)$$

where V_t^U denotes the unlevered firm value, r_U the cost of equity of an unlevered firm, r_D the cost of debt and $E_t[\cdot]$ the expected value operator under the subjective probability measure \mathbb{P} conditional on the available information in period t . For simplicity purposes we assume that the corporate tax rate τ_C , the unlevered cost of equity r_U and the cost of debt r_D are constant. Under the risk-neutral probability measure \mathbb{Q} the levered firm value can be equivalently determined via

$$V_t^L = \sum_{s=t+1}^T \frac{E_t^Q[FCF_s^U]}{(1+r_f)^{s-t}} + \sum_{s=t+1}^T \frac{E_t^Q[TS_s]}{(1+r_f)^{s-t}}, \quad (4)$$

where $E_t^Q[\cdot]$ is the expectation operator under the risk-neutral probability measure \mathbb{Q} conditional on the available information in t and r_f denotes the constant risk-free rate. Note that in this case by only considering corporate taxes τ_C the period-specific tax savings TS_s can be explicitly expressed by $\tau_C \cdot r_f \cdot D_{s-1}$. Assuming a perpetual debt level, i.e. debt stays constant $D_t = D_{t+1} = \dots = D_T$, we get the classic result that the present value of the tax savings simplifies to $\frac{\tau_C \cdot r_D \cdot D_t}{r_D} = \tau_C \cdot D_t$.

By considering personal taxes on payments to equity holders as dividends which are taxed on the personal level with a tax rate τ_P , the tax shield increases to $[\tau_C + (1 - \tau_C) \cdot \tau_P] \cdot r_D \cdot D_t$. While interest payments on a corporate level reduce corporate and personal taxes, the interest income is taxed at the personal level with a tax rate τ_D , which in turn decreases the tax savings to $[\tau_C + (1 - \tau_C) \cdot \tau_P - \tau_D] \cdot r_D \cdot D_t$. In an extreme scenario, the taxation of interest income on a personal level could exceed the tax advantage of debt. Therefore the marginal benefit of one unit interest payment instead of one equity payout amounts to

$$(1 - \tau_D) - (1 - \tau_C) \cdot (1 - \tau_P). \quad (5)$$

This implies that the levered firm value considering personal taxes on dividends τ_P and interest payments τ_D as well as the after-tax cost of debt $r_D \cdot (1 - \tau_D)$ for discounting the future tax savings is determined by (see for example Miller (1977), Graham (2003) or van Binsbergen et al. (2010))

$$V_t^L = V_t^U + \sum_{s=t+1}^T \frac{E_t\left[\left[(1 - \tau_D) - (1 - \tau_C) \cdot (1 - \tau_P)\right] \cdot r_D \cdot D_{s-1}\right]}{\left[1 + r_D \cdot (1 - \tau_D)\right]^{s-t}}. \quad (6)$$

This pricing equation can be easily transferred into a model utilizing the risk-neutral probability measure.

2.2. Modelling intercompany financing

The standard approach for valuing tax shields determines the tax shield on a single firm basis. In order to be able to analyze the impact of a thin-capitalization rule and for highlighting the differences to an earnings stripping rule, we have to extend the single firm framework by considering the interrelations on group level, for example the relation between a parent company and an exemplary subsidiary with respect to debt financing. Figure 1 depicts the parent-subsidiary framework for our model analysis. The overall debt level of the subsidiary amounts in an arbitrary period t to D_t^{sub} . The subsidiary borrowed a proportion of α , with $\alpha \in [0, 1]$, of its total debt from an external debtholder and $1 - \alpha$ from the subsidiary's parent company. For simplicity purposes we assume that the applicable cost of debt r_D are the same for both debt issues. Consequently, the total interest payments of the subsidiary amount to $I_{t+1}^{sub} = r_D \cdot D_t^{sub}$ and by applying the respective proportions the interest payments distributed to the external debtholders, $\alpha \cdot I_{t+1}^{sub}$, as well as the interest payments for the parent company, $(1 - \alpha) \cdot I_{t+1}^{sub}$, can be obtained. The subsidiary's after-tax income is completely distributed to the parent company and afterwards to the individual investor. In addition, we assume that the parent company has in an arbitrary period t a debt level of D_t^{par} and pays interest payments of $I_{t+1}^{par} = r_D \cdot D_t^{par}$ to external debtholders.

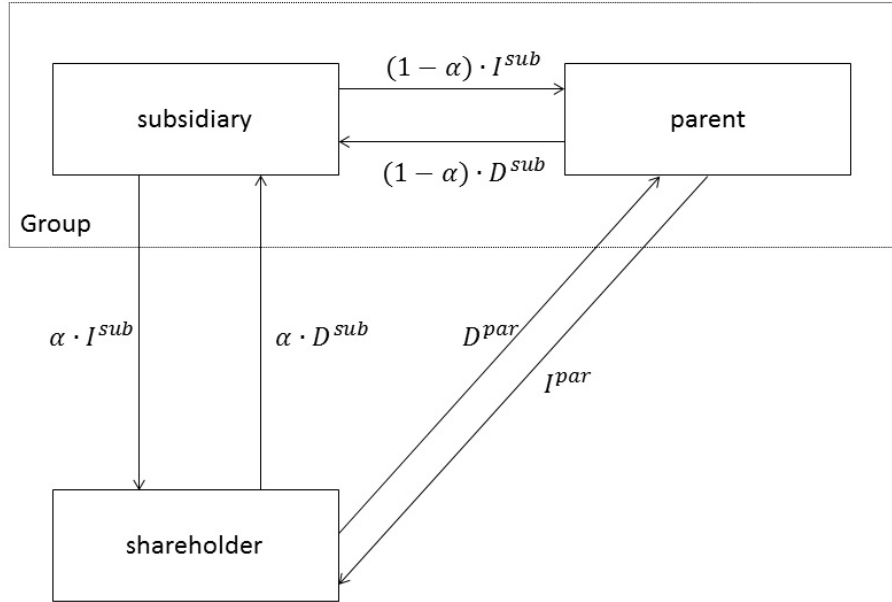


Fig. 1
Relationship between Parent and Subsidiary.

This figure depicts the financing relationship between the parent and the subsidiary. The subsidiary finances a proportion α of its overall debt D^{sub} from an external claim holder and $(1 - \alpha)$ from its parent company. Accordingly, the interest payments $(1 - \alpha) \cdot I^{sub}$ are paid to the parent and $\alpha \cdot I^{sub}$ to the external claim holder. The parent might finance its operations partly by debt with an amount of D^{par} and pays interest I^{par} .

For analyzing the overall impact of a thin-capitalization rule on the tax shield value on an individual level, we have to assume that the shareholders of the parent company are as well the debtholders under consideration. In order to avoid an indirect substantial participation of the shareholders in the subsidiary, we additionally assume that the subsidiary is fully owned by the parent and that the number of free float shares of the parent amounts to 100%.

Thin-capitalization and earnings stripping rules target on the limitation of extensive debt financing. A (consistent) model that aims at mapping these tax rules additionally has to deal with thin capitalized, or more precisely with highly levered firms. Since we model the unlevered free cash flows by a recombining binomial tree, the implied equity value in a specific state ω and an arbitrary period t is determined by $E_t(\omega) = V_t^L(\omega) - D_t$ might become negative (dependent on the state-dependent free cash flow). This would usually constitute a default due to indebtedness, i.e. $E_t(\omega) < 0$. One possibility would be to assume that the firm immediately goes bankrupt. The debtholders could take over the firm and either liquidate the remaining assets, sell the overall firm to a new investor or reorganize the firm to carry debt again. Independent of the new owner of the firm, one of the standard assumptions within this context is that the tax shield vanishes after default has occurred (see e.g. Cooper and Nyborg (2008) or Koziol (2014)). Another possibility could be to keep the firm running and prevent the default estate by injecting new equity. In the context of intragroup financing the parent company would inject a new amount of equity into the subsidiary. The injected amount offsets the negative equity value and up to a lower equity level of E_t^{inj} . We will base our subsequent analysis on the latter assumption and thereby exclude the possibility of default.

Without any further explanation, the exclusion of default through an equity injection by the parent might not be reasonable. In general, an equity injection should be modelled depending on its feasibility, i.e. an equity investor would only make an (additional) equity investment if the net present value of the investment opportunity would be positive. In case of a negative net present value the equity investor would not inject new equity and the firm would default. However, in order to perform a selective analysis of the thin-capitalization and earnings stripping rules, the assumption that the parent injects in any case new equity enables us to independently quantify the effects of these tax regulations on the overall tax shield value.

2.3. Tax shield valuation without interest limitation rules

The classical approach for valuing tax shields builds upon the premise that interest payments are always tax deductible. In order to provide a condensed notation and clear differentiation we use superscripts denoting the different cases. In case without any interest limitation rule (N) the overall tax savings with parent-subsidiary financing amount in an arbitrary period $t + 1$ to

$$TS_{t+1}^N = \left(I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub} \right) \cdot (1 - \tau_D) - (1 - \tau_C) \cdot (1 - \tau_P) \cdot \left[I_{t+1}^{par} + (\alpha - y \cdot \tau_C) \cdot I_{t+1}^{sub} \right], \quad (7)$$

where y denotes the taxable portion of dividend income. Dividend payments distributed from one firm to another is typically exempted from taxation³ to 95% or 100% or stated differentially a maximum of 5% of the paid dividend is subject to taxes on the parent level.

The tax shield resulting from the parent-subsidiary relation arises due to the following facts: (1.) The subsidiary's interest expenses result in a decreasing distribution to the parent. This reduces the tax burden by $-y \cdot \tau_C$. (2.) The tax relief of internal interest expenses at the subsidiary's level amounting to $-\tau_C$ leads to interest income on the parent level and thereby the tax burden rises by τ_C . Hence, on the overall group level the tax shield derives from the double-taxation of dividends. Additionally, note that the country specific tax rates (τ_D , τ_P and τ_C) (and below without forestalling, the respective terms for the thin-capitalization rules) have to be plugged-in ex ante to derive the tax shield formula.

³We can consider this, since we assumed that the parent is participated to 100% in the subsidiary. In some countries, e.g. Austria, Belgium, Finland, France, Portugal and most recently Germany (since March 1, 2013), the participation exemption is granted for dividend income with a minimum participation of 10% or 5% (France).

3. Thin-Capitalization Rules

3.1. Tax shield valuation with thin-capitalization rules

Even though several approaches (for example Molnár and Nyborg (2013), Couch et al. (2012) or Koziol (2014)) control for a possible loss of future tax shields due to a possible default, in most tax jurisdictions the tax deductibility of interest payments of the overall firm or group might be limited even before a possible default occurs. Depending on the tax regulation of the jurisdiction either earnings stripping or thin-capitalization rules have been established to limit the tax deductibility of extensive debt financing. While the latter have already a long tradition, in several tax codes the earnings stripping rules are a relatively new trend⁴ for limiting the tax deductibility of interest payments. Thin-capitalization rules usually limit to a certain extent the tax deductibility of intragroup interest payments, i.e. interest payments on debt that has been provided from the parent to a certain subsidiary. In contrast to that, earnings stripping rules are based upon the total interest payments of any company, which implies that the interest expenses are limited independent of a parent-subsidiary relation. For explicitly showing the impact of each of the rules on tax shield valuation, we focus in this section on thin-capitalization rules and expand our analysis towards earnings stripping rules in section 4.

Throughout our analysis in this section, we will show the impact of the thin-capitalization rules on tax shield valuation that are predominant in the EU-15 countries: Austria (AUT), Belgium (BEL), Denmark (DEN), Finland (FIN), France (FRA), Germany (GER), Greece (GRE), Italy (ITA), Ireland (IRL), Luxembourg (LUX), Portugal (POR), Spain (ESP), Sweden (SWE) and United Kingdom (UK). As already outlined in the introduction, we focus on this specific country group due to the impact of the well-known Lankhorst-Hohorst decision⁵ by the ECJ in 2002 which became effective on January 1, 2004.

As shown in Table 2, the explicit treatment of the non-deductible part of the interest payments implied by the thin-capitalization rule depends on the country-specific tax code. In some countries the non-tax-deductible internal interest expense is reclassified as dividend paid from the subsidiary to the parent, while in others this is not the case. Due to the fact that on the parent level dividend income is only with a portion of y whereas interest income is entirely subject to taxes, we have to differentiate between the reclassification (rec) and no-reclassification (nor) case.

In the nor-case we observe two effects: (1.) Since a proportion of the overall interest payments is non-deductible for tax purposes, the tax payments increase on the subsidiary's level and therefore (2.) decrease the distributions to the parent. Nevertheless, on the parent level these payments are fully taxable since these can be regarded as interest income, which in turn implies an overall decrease of the distributions to the shareholders of the parent. In the rec-case as in the nor-case, the non-tax-deductible interest payments imply a higher tax payment by the subsidiary. But in contrast to the nor-case, the reclassification of the non-tax-deductible part of the internal interest payments implies that the taxes paid by the parent are smaller as in the nor-case, due to the fact that the reclassified interest income is almost tax exempted.

To determine the excessive internal debt, most countries define a specific debt-to-equity ratio which we will denote by $\frac{D^{TC}}{E^{TC}}$. The interest payments on the debt amount that exceeds $\frac{D^{TC}}{E^{TC}}$ are subject to the respective thin-capitalization rule and therefore not tax-deductible. Since the subsidiary's considered debt and equity amount varies across countries between total, only internal, individual internal or internal foreign debt or equity, the applicable debt and equity is determined via $\frac{h \cdot D^{sub}}{j \cdot E^{sub}}$.

Table 2 provides an overview of the current thin-capitalization rules in the EU-15 countries and demonstrates the tax shield including thin-capitalization rules. For the analysis of thin-capitalization rules, we (still) rely on the standard assumptions of the APV approach by assuming that the cost of equity of an unlevered firm r_U , the cost of debt r_D , the risk free rate r_f and the tax rates for personal taxes

⁴As depicted in Table 1 the German tax legislative was among the first in the EU-15 to implement such a rule. In the USA the possible introduction of an earnings stripping rule resulted in a broad public discussion.

⁵ECJ, C-J032/00.

Table 2
Country-specific Thin-Capitalization Rules in the EU-15 Countries

Country	AUT	BEL	DEN ²	FIN ¹	FRA ¹	GER ¹	GRE ¹	IRL	ITA ¹	LUX	NED ¹	POR ¹	ESP ¹	SWE	UK
<i>Thin-capitalization rule</i>	no	yes	yes	no	yes	yes	no	no	yes	no	yes	yes	yes	no	yes
<i>Kind of shareholder</i>		all	all		all	all			all		all	non EU	non EU		all
<i>Tax rates</i>															
<i>Corporate tax rate</i> τ_C	25.00%	33.99%	24.50%	20.00%	36.40%	30.18%	26.00%	12.50%	27.50%	29.22%	25.00%	31.50%	30.00%	22.00%	21.00%
<i>Dividend tax rate</i> τ_P	25.00%	25.00%	42.00%	27.20%	44.00%	26.38%	10.00%	48.00%	20.00%	20.00%	25.00%	28.00%	27.00%	30.00%	30.56%
<i>Interest tax rate</i> τ_D	25.00%	25.00%	51.50%	30.00%	44.00%	26.38%	40.90%	48.00%	20.00%	10.00%	30.00%	25.00%	27.00%	30.00%	50.00%
<i>Substantial participation</i>			50.00%		50.00%	25.00%			25.00%		33.00%	10.00%	25.00%		75.00%
<i>Applicable debt-to-equity ratio</i> $B = \frac{D^TC}{E^TC}$		5:1	4:1		1.5:1	1.5:1	3:1		4:1		3:1	2:1	3:1		1:1 ³
<i>Safe haven</i>															
	$H_{t+1} = (1 - \alpha) \cdot \tau_D \cdot E_t^{sub} \cdot \frac{D^TC}{E^TC}$														
<i>Tax shield with thin-capitalization</i>															
	<i>reclass. as dividends</i> (BEL, GER, ITA, ESP)	$T_{t+1}^{rec} = \left(\frac{p^{par}}{t_{t+1}} + \alpha \cdot i_{t+1}^{sub} \right) \cdot (1 - \tau_D) - (1 - \tau_C) \cdot \left[\frac{p^{par}}{t_{t+1}} + (1 - y \cdot \tau_C) \cdot \alpha \cdot i_{t+1}^{sub} - y \cdot \tau_C \cdot H_{t+1} \right]$													
	<i>no reclass. as dividends</i> (DEN, FRA, NED, POR, UK)	$T_{t+1}^{nor} = \left(\frac{p^{par}}{t_{t+1}} + \alpha \cdot i_{t+1}^{sub} \right) \cdot (1 - \tau_D) - (1 - \tau_P) \cdot \left[(1 - \tau_C) \cdot \frac{p^{par}}{t_{t+1}} + [(1 - \alpha \cdot \tau_C) \cdot (1 - y \cdot \tau_C) - (1 - \tau_C)] \cdot i_{t+1}^{sub} - (1 - y \cdot \tau_C) \cdot \tau_C \cdot H_{t+1} \right]$													

¹ Earnings stripping rule since 2008 (GER, ITA), 2012 (ESP), 2013 (FRA, NED, POR), 2014 (FIN, GRE).

² Thin-capitalization rules apply only if an exemption threshold of DKK10 million is exceeded.

³ Generally accepted arm's-length principle.

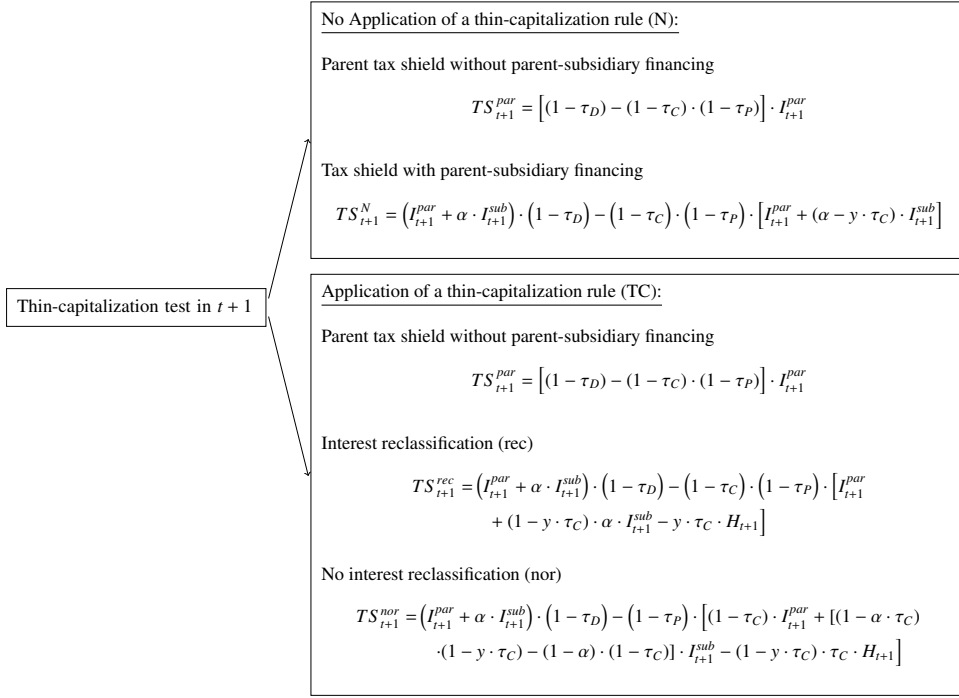


Fig. 2
Structure of Thin-Capitalization Rules.

This figure shows the tax shield on the level of the individual investor with parent-subsidiary financing, for the cases without and with the application of a thin-capitalization rule. In the latter we differentiate between the two cases interest reclassification (rec) and no interest reclassification (nor). Note that in the prior period no default has occurred.

on interest income τ_D , dividends τ_P as well as the corporate income tax τ_C are constant. In general, a country-specific thin-capitalization rule can become effective if the current debt-to-equity ratio exceeds $\frac{D^{TC}}{E^{TC}}$ according to Table 2 is applicable.

Figure 2 depicts the general setting for the application of an arbitrarily defined thin-capitalization rule. In the case where it applies (TC), the period-specific tax savings depend on whether the respective tax authority reclassifies interest paid from the subsidiary to the parent as dividends (rec) or not (nor). Table 2 also provides an overview on the country-specific reclassification schemes. However, with respect to the tax savings in the case where the thin-capitalization rule applies, we may write down for countries with interest reclassification

$$TS_{t+1}^{rec} = (I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub}) \cdot (1 - \tau_D) - (1 - \tau_C) \cdot (1 - \tau_P) \cdot [I_{t+1}^{par} + (1 - y \cdot \tau_C) \cdot \alpha \cdot I_{t+1}^{sub} - y \cdot \tau_C \cdot H_{t+1}] \quad (8)$$

and for countries without interest reclassification

$$TS_{t+1}^{nor} = (I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub}) \cdot (1 - \tau_D) - (1 - \tau_P) \cdot [(1 - \tau_C) \cdot I_{t+1}^{par} + [(1 - \alpha \cdot \tau_C) \cdot (1 - y \cdot \tau_C) - (1 - \alpha) \cdot (1 - \tau_C)] \cdot I_{t+1}^{sub} - (1 - y \cdot \tau_C) \cdot \tau_C \cdot H_{t+1}]. \quad (9)$$

Since we have already differentiated the thin-capitalization rules into a rec- and a nor-case for mapping the possible interest payments reclassification as dividends, the so called safe haven H remains besides the respective tax rate as an important country-specific determinant. It determines the maximum tax-deductible interest payments in an arbitrary period $t + 1$ and is in its most general form in our model given

by

$$H_{t+1} = (1 - \alpha) \cdot r_D \cdot D_t^{sub} \cdot \frac{j}{h} \cdot \frac{E_t^{sub}}{D_t^{sub}} \cdot \frac{D^{TC}}{E^{TC}}, \quad (10)$$

where $\frac{E_t^{sub}}{D_t^{sub}}$ represents the inverse debt-to-equity ratio of the subsidiary in an arbitrary period t and $\frac{D^{TC}}{E^{TC}}$ the ratio $\frac{D^{TC}}{E^{TC}}$ according to the country-specific thin-capitalization rule. By simplifying and noting that $D_t^{TC} = \frac{j}{h} \cdot E_t^{sub} \cdot \frac{D^{TC}}{E^{TC}}$ represents the total amount of debt whose interest payments are tax deductible, H_{t+1} simplifies to

$$H_{t+1} = (1 - \alpha) \cdot r_D \cdot D_t^{TC}. \quad (11)$$

As long as the debt-to-equity ratio falls short of $\frac{D^{TC}}{E^{TC}}$, the interest payments remain fully tax deductible. If the current debt-to-equity ratio exceeds $\frac{D^{TC}}{E^{TC}}$, equation (11) determines the maximum tax-deductible interest payments. Therefore the relation of $\frac{D_t^{sub}}{E_t^{sub}}$ to $\frac{D^{TC}}{E^{TC}}$ determines the calculation of the period-specific tax savings: Either the tax savings are determined via equation (7) or depending on a possible reclassification by equation (8) or (9).

With this explicit modelling, it suffices for finding a general expression for the tax shield value to distinguish between a general case representing the application of a thin-capitalization rule TS_{t+1}^{TC} and a case without the application TS_{t+1}^N .

By following Appendix A we note that $TS_{t+1}^N > TS_{t+1}^{TC}$ does not hold for all values of D_t^{sub} . This directly implies that the period-specific tax savings in an arbitrary period t are determined by

$$TS_{t+1} = \min(TS_{t+1}^N, TS_{t+1}^{TC}) \quad (12)$$

or equivalently

$$TS_{t+1} = TS_{t+1}^N - \max(TS_{t+1}^N - TS_{t+1}^{TC}, 0). \quad (13)$$

In the following, we derive a model for evaluating the impact of a possible application of a thin-capitalization rule by using a recombining binomial lattice. The implementation is highly interrelated to the design of the country-specific regulation but can be simplified to a more general form which remains applicable by using the specifications of each country according to Table 2.

Under consideration of the risk-neutral probability measure \mathbb{Q} , the value of the tax savings in t is in any case given by

$$VTS_t = \sum_{s=t+1}^T \frac{E_t^{\mathbb{Q}} \left[TS_s^N - \max(TS_s^N - TS_s^{TC}, 0) \right]}{(1 + r_f)^{s-t}}. \quad (14)$$

By the rules of conditional expectations we may write equivalently

$$VTS_t = \underbrace{\sum_{s=t+1}^T \frac{E_t^{\mathbb{Q}} [TS_s^N]}{(1 + r_f)^{s-t}}}_{VTS_t^N} - \sum_{s=t+1}^T \frac{E_t^{\mathbb{Q}} [\max(TS_s^N - TS_s^{TC}, 0)]}{(1 + r_f)^{s-t}}. \quad (15)$$

The first term represents the normal tax shield value according to equation (6) under the risk-neutral

probability measure. The second term is an option-like payoff that depends on the specifications of the respective tax code. Regardless of the reclassification treatment, the function TS_{t+1}^{TC} depends via the equation for the safe haven on the equity value E_t^{sub} of the levered firm.

More explicit versions of the tax shield value equation can be obtained by substituting for TS_{t+1}^{TC} the respective equations for the rec- (8) or nor-case (9). In the first one the maximum function in equation (15) is given in an arbitrary period by $\max(TS_{t+1}^N - TS_{t+1}^{rec}, 0)$. By using equation (7) and (8) and rearranging we get for the value of the tax shield

$$VTS_t^{rec} = \sum_{s=t+1}^T \frac{E_t^Q[TS_s^N]}{(1+r_f)^{s-t}} - \sum_{s=t+1}^T \frac{(1-\tau_C) \cdot (1-\tau_P) \cdot y \cdot \tau_C \cdot E_t^Q[\max((1-\alpha) \cdot I_s^{sub} - H_s, 0)]}{(1+r_f)^{s-t}}. \quad (16)$$

The first fraction yields the regular tax shield value without limitation. The second fraction depicts the tax shield reduction caused by a thin-capitalization rule in the rec-case. As non-deductible internal interest expenses are reclassified as dividends, the term $-(1-\tau_C) \cdot (1-\tau_P) \cdot y \cdot \tau_C$ expresses the additional taxation on (increased) dividends at the investors level. The term $\max((1-\alpha) \cdot I_s^{sub} - H_s, 0)$ represents the non-deductible part of internal interest expenses which can reach a maximum value of I_s^{sub} for $\alpha = 0$. In that case no internal interest expenses are tax-deductible since these are reclassified as dividends. The tax saving, which results from avoiding the double taxation of the subsidiary's taxable income with τ_C at the subsidiary's and $y \cdot \tau_C$ at the parent company's level, is eliminated.

By performing an equivalent substitution for the nor-case, we get a slightly different tax shield valuation equation

$$VTS_t^{nor} = \sum_{s=t+1}^T \frac{E_t^Q[TS_s^N]}{(1+r_f)^{s-t}} - \sum_{s=t+1}^T \frac{(1-y \cdot \tau_C) \cdot (1-\tau_P) \cdot \tau_C \cdot E_t^Q[\max((1-\alpha) \cdot I_s^{sub} - H_s, 0)]}{(1+r_f)^{s-t}}. \quad (17)$$

The second fraction represents the tax shield reduction in the nor-case. The term $-(1-y \cdot \tau_C) \cdot (1-\tau_P) \cdot \tau_C$ describes the additional taxes as the internal interest expenses are not deductible at the investors level. When no internal interest expenses are deductible, the tax shield reaches a negative value of $-(1-y) \cdot (1-\tau_P) \cdot \tau_C \cdot I_s^{sub}$. On the one hand, the tax payments on the subsidiary's level arise as interest expenses are not deductible ($-\tau_C$). On the other hand, the taxation of the dividends - which is only taxed by $y \cdot \tau_C$ - decrease as the distribution to the parent diminishes. The tax shield becomes negative as the interest income of the parent is still taxed at a rate of τ_C irrespective of the deduction at the subsidiary's level. The decreasing distribution to the parent can only slightly cover the negative aspects.

Note that the equations (16) and (17) only differ with respect to the term in front of the maximum function. As a direct consequence of the dependency on the safe haven H_s and in turn on the state-dependent equity value of the subsidiary E_t^{sub} , we have to track the equity values for all future states. While this matter can be easily mapped in a binomial lattice, the interdependency of E_t^{sub} in an arbitrary period and state from the tax shield value according to $E_t = V_t^U + VTS_t - D_t$ complicates matters (see Figure 3). We can observe the tax shield depends on the equity value and vice versa. Nevertheless, we overcome this circularity problem by bisection.

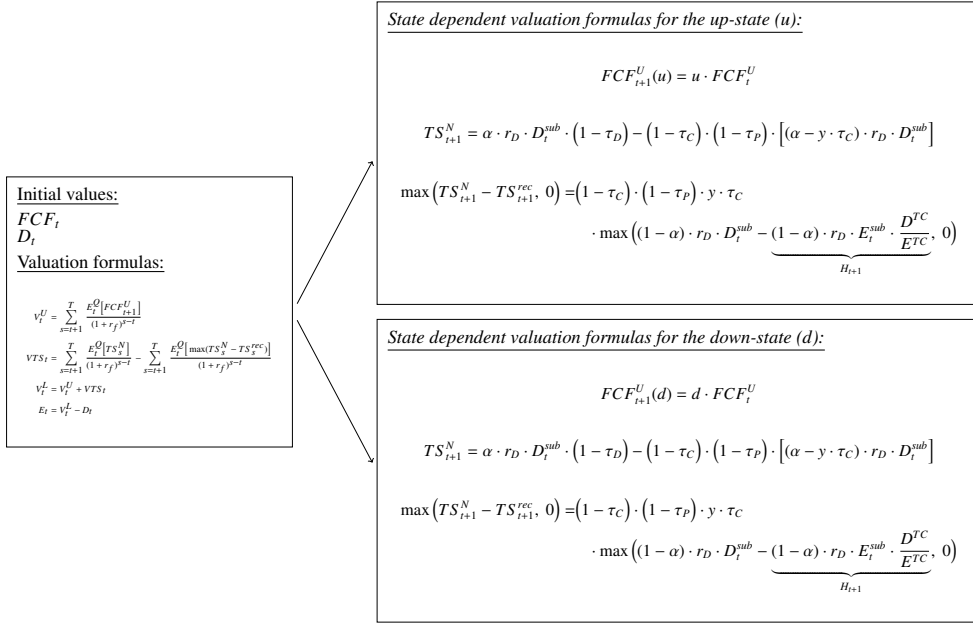


Fig. 3

The Circularity of Thin-Capitalization Rules - A two Binomial Step Example using the Rec-case.

This figure depicts the circularity of the thin-capitalization rule according to the reclassification case (rec) in a two period binomial tree. This tree can easily be adjusted for the nor-case by using the respective equations. For shortening notation we set $I_{t+1}^{par} = 0$. With this figure we highlight that the maximum function in both states, up (u) and down (d), in period $t + 1$ depends on the safe haven H_{t+1} and in turn on the equity value in period t . At the same time the equity value is determined via the well-known equation $E_t = V_t^U + VTS_t - D_t$ and therefore, depends on the value of the tax shield. This circularity can be easily overcome by bisection.

3.2. Numerical example: thin-capitalization rules

In this subsection, we show the dynamics of the thin-capitalization rules according to the rec- and the nor-case in a numerical example by using as basis the aforementioned recombining binomial tree for modelling the evolvement of the unlevered free cash flows. While the recombining feature of the unlevered free cash flows and firm remains intact, the tax shield subject to a possible application of the thin-capitalization rule and in turn the equity value becomes path-dependent. As basis parameters for the recombining binomial lattice we assume an initial free cash flow (FCF_t^U) of 100 an up-factor $u = 1.4$ ($d = 1/1.4$)⁶, and equal up- and down-probabilities with $q = 0.5$. The risk-free interest rate is set accordingly to 5.714%.

In order to highlight the impact of the thin-capitalization rule on the overall tax shield value, we assume that the parent has no debt outstanding ($D^{par} = 0$) and set $\alpha = 0$. The subsidiary performs a constant debt level policy with $D^{sub} = 700$ ($I^{sub} = 40$) which implies a leverage ratio in terms of D/V^U in period t of 58.33. This debt level has been chosen in order to get as result a clear differentiation between states in which the thin-capitalization rule is not and is applicable. As parameters describing the tax jurisdiction Italy we assume for both cases the following: $\frac{D^{TC}}{E^{TC}} = 4$, $\tau_C = 27.5\%$, $\tau_P = 20\%$, $\tau_D = 20\%$, $j = 1$ and $h = 1$. In case of equity values lower than $E_t = 10$, new equity is injected up to a level of $E_t^{inj} = 10$.

⁶An up-factor of $u = 1.4$ implies an annual volatility of 33.65% which is an appropriate assumption for the standard deviation of the free cash flows of a listed firm in the EU-15.

As demonstrated in Figure 4 the present value of the tax shield generated by the subsidiary's debt on the level of the individual investor without considering an application of interest limitation rules amounts to $VTS_t^N = 5.583$ (in a single period $TS_s^N = 0.319$). By considering the consequences of a thin-capitalization rule in the rec-case the overall tax shield is given by $VTS_t^{rec} = 5.583 - 0.098 = 5.485$. This small impact of the thin-capitalization rule is subject to the fact that non-deductible interest expenses are reclassified as dividends which are subject to tax. In order to illustrate this, we focus on the *uu*- and *ud*-state. In the *uu*-state, the maximum tax-deductible interest amount (10) in our model amounts to $H_{t+2} = (1 - 0) \cdot (0.05714 \cdot 700 \cdot \frac{1}{1} \cdot \frac{845.583}{700} \cdot \frac{4}{1}) = 193.266$ (while the complete interest expenses amount to $I_{t+2}^{sub} = 40$). The safe haven debt-to-equity ratio is not exceeded $\frac{700}{845.583} - \frac{4}{1} = -3.172$ and all interest expenses are deductible. As the safe haven exceeds the interest expenses in the specific period, the tax shield for the rec-case $TS_{t+2}^{rec} = (1 - 0.275) \cdot (1 - 0.2) \cdot (0.05 \cdot 0.275 \cdot 193.266) = 1.541$ is higher than the tax shield without interest limitation. In this case $TS_{t+2}^N < TS_{t+2}^{rec}$ and the function $\max(TS_{t+2}^N - TS_{t+2}^{rec}, 0)$ results in $\max(0.319 - 1.541, 0) = 0$. The thin-capitalization rule does not apply and the tax shield value according to equation (13) amounts to $TS_{t+2} = 0.319$. In contrast in the *dd*-state, the safe haven debt-to-equity ratio is exceeded by $\frac{700}{91.017} - \frac{4}{1} = 3.691$ and so the safe haven amounts to $H_{t+2} = 20.804$. As $TS_{t+2}^{rec} = 0.166$, the maximum function amounts to $\max(0.319 - 0.166, 0) = 0.153$ and the final tax shield is reduced to $TS_{t+2} = 0.319 - 0.153 = 0.166$. The outstanding interest expenses are not deductible at the subsidiary's level and reclassified at the parent level. As the implied equity value gets negative in the specific case ($E_{t+2} = -191.954$), we assume that it gets injected up to 10.

For the analysis of the nor-case we use the same parameters as aforementioned. As shown in Figure 5 the tax shield considering this respective thin-capitalization rule amounts to $VTS_t^{nor} = 5.583 - 3.766 = 1.817$. In comparison to the rec-case the non-deductible part of the subsidiary's internal interest expenses are fully taxable at the parent level resulting in an overall smaller tax shield value. To exemplify with the *dd*-state the equivalent values amount to $H_{t+2} = 19.0481$, $TS_{t+2}^{nor} = -(1 - 0.2) \cdot [(0.275 - 0.05 \cdot 0.275) \cdot 40 - (1 - 0.05 \cdot 0.275) \cdot 0.275 \cdot 19.048] = -0.8 \cdot [10.450 - 5.166] = -4.227$ and $\max(0.319 - (-4.227), 0) = 4.546$. In the specific period the tax shield $TS_{t+2} = 0.319 - 4.546 = -4.227$ gets negative. The tax savings of the deductible interest expenses $(-(1 - 0.2) \cdot 0.05 \cdot 0.275 \cdot (1 - 0.275) = -0.152)$ do not cover the additional taxation of the non-deductible interest expenses $((1 - 0.2) \cdot (1 - 0.05) \cdot 0.275 = 4.379)^7$ anymore.

⁷See for an explanation Appendix A.

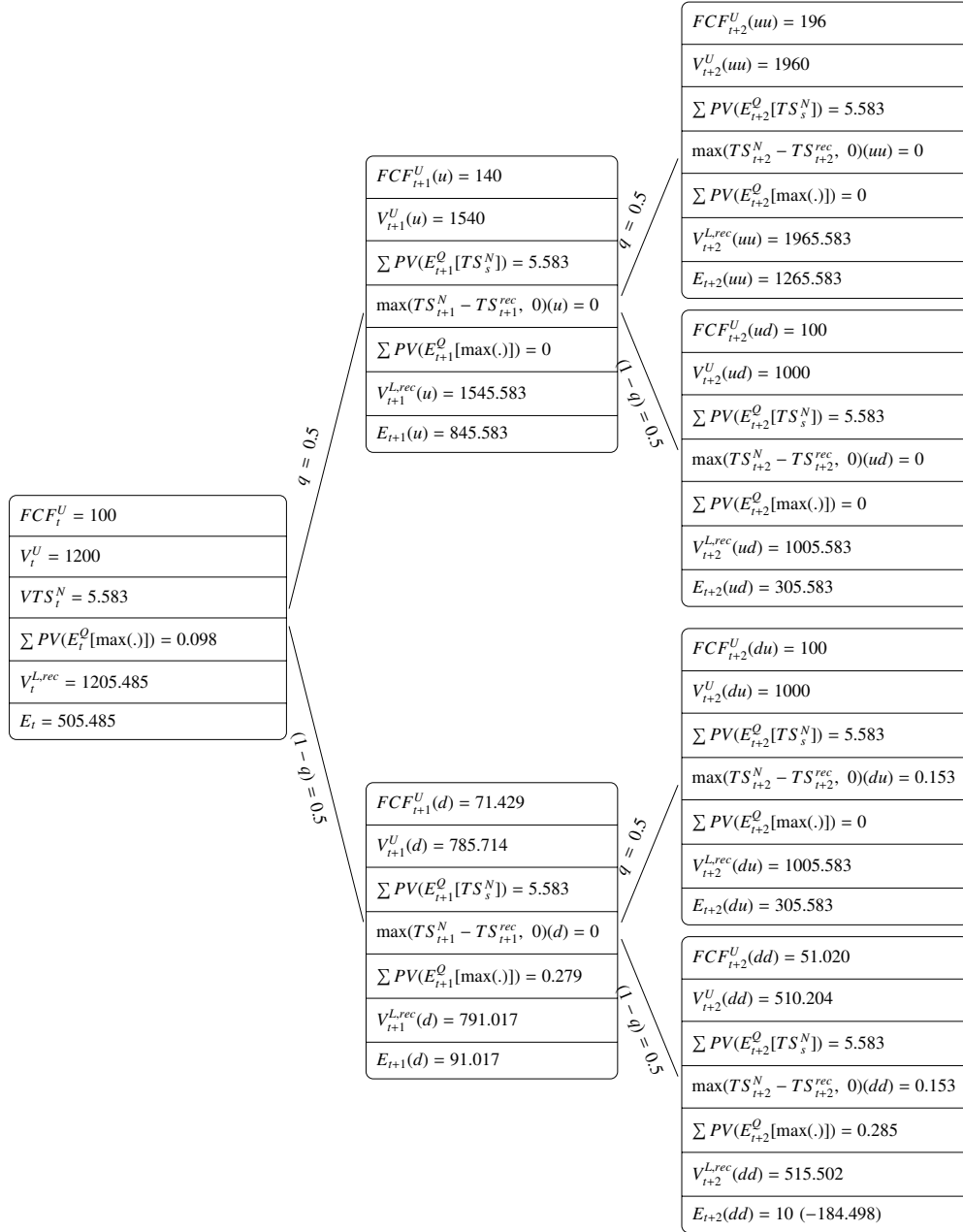


Fig. 4
Numerical Example for a Thin-Capitalization Rule in the Rec-Case.

Throughout this numerical example we use the parameters as given in section 3.2. For providing a clear figure we have abstained from depicting periods with $s > t + 2$. From period $t + 4$ onwards we have assumed for ease of calculation a perpetual tax shield of 5.583. We denote the different state dependent quantities by the respective up- or down-movements, i.e. the free cash flow in period $t + 2$ resulting from one up- and one down-movement is denoted by $FCF_{t+2}^U(ud)$. The results of all calculations are rounded to four digits. For completeness and plausibility we show in case of indebtedness the implied negative equity value. Obviously, in case of a limited liability firm the equity value is then zero; concerning our assumptions it is injected up to 10.

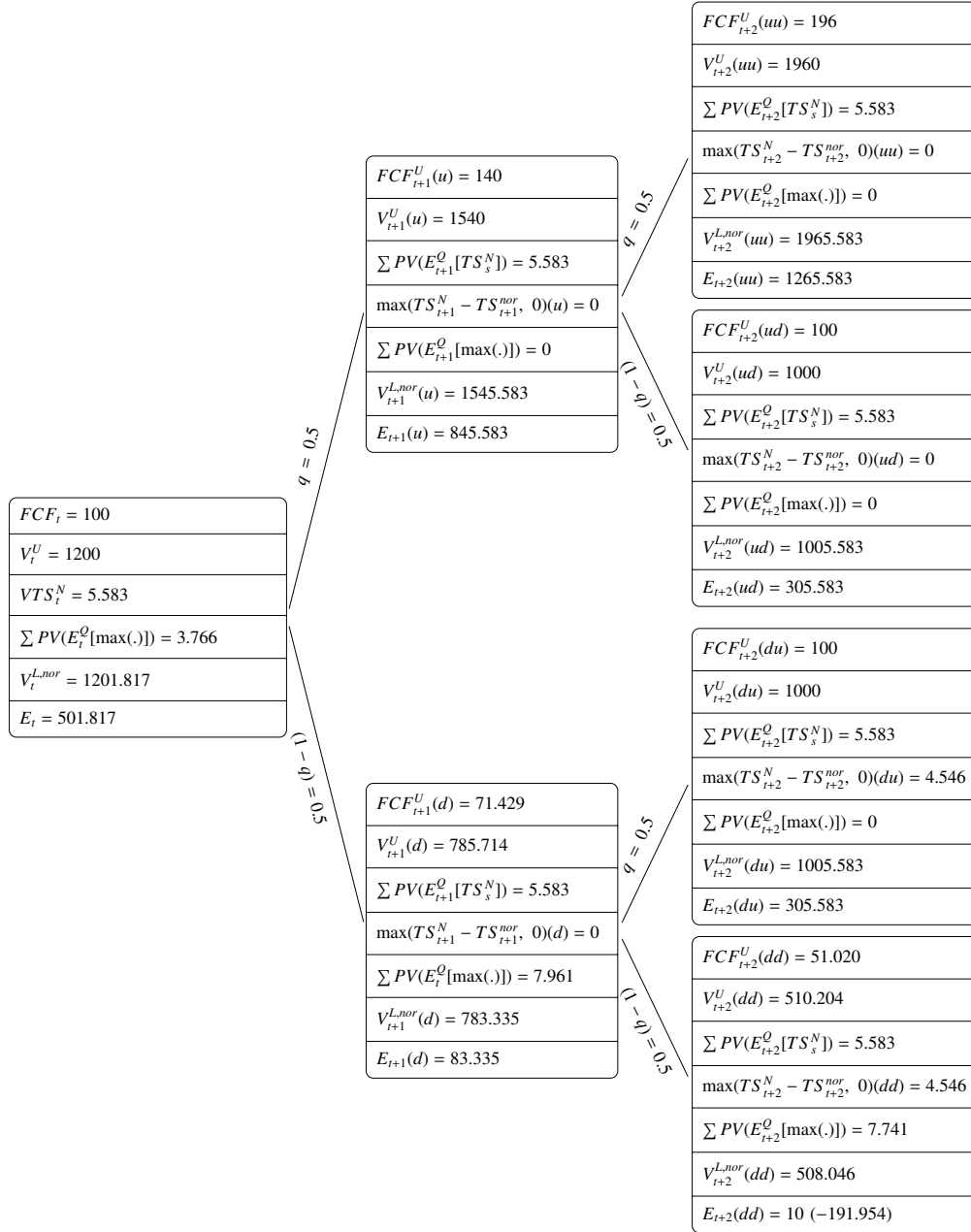


Fig. 5
Numerical Example for a Thin-Capitalization Rule in the Nor-Case.

Throughout this numerical example we use the parameters as given in section 3.2. For providing a clear figure we have abstained from depicting periods with $s > t + 2$. From period $t + 4$ onwards we have assumed for ease of calculation a perpetual tax shield of 5.583. We denote the different state dependent quantities by the respective up- or down-movements, i.e. the free cash flow in period $t + 2$ resulting from one down- and one up-movement is denoted by $FCF_{t+2}^U(du)$. The results of all calculations are rounded to three digits. For completeness and plausibility we show in case of indebtedness the implied negative equity value. Obviously, in case of a limited liability firm the equity value is then zero; concerning our assumptions it is injected up to 10.

Throughout section 5, we show the impact of different debt levels and their implied application of the thin-capitalization rule on the overall tax shield value.

4. Earnings Stripping Rules

4.1. Tax shield valuation with earnings stripping rules

Since 2008, some of the EU-15 countries, i.e. Germany, Italy, Portugal, Spain and in 2014 Finland and Greece, introduced or replaced their thin-capitalization rules by earnings stripping rules. According to the German model, which can be regarded as a kind of role model⁸, this limitation applies in general to all interest expenses of a firm, whether the debt is granted by a third party, related party or a shareholder. The deductibility of annual net interest expense $r_D \cdot D_t$ ⁹ (interest expense exceeding interest income) is limited to a certain percentage β of the earnings before interest, tax, depreciation and amortization (EBITDA), if the debt value is above a certain threshold. This debt threshold ranges from 0.5 million EUR in Finland to 5 million EUR in Greece and is assumed to be exceeded in our framework.

The non-deductible interest expenses IC_{t+1} can be carried forward and subtracted in future periods. Therefore, there might be a positive influence on the tax shield in future periods as there is a higher deductible interest potential. In Portugal and Spain the interest carryforward is limited to 5 and 18 years. Without time restriction and disregarding negative EBITDA values it is determined by

$$IC_{t+1} = \max(r_D \cdot D_t + IC_t - \beta \cdot EBITDA_t, 0). \quad (18)$$

Thus, the general deductible interest amount is given by

$$\Psi_{t+1} = \min(\beta \cdot EBITDA_t, r_D \cdot D_t + IC_t). \quad (19)$$

Within the framework of the Growth Acceleration Act (Wachstumsbeschleunigungsgesetz)¹⁰, Germany on the one hand increased the certain threshold from 1 to 3 million EUR and on the other hand introduced the possibility to carry forward (for 5 years) the amount of the EBITDA which is not used for deduction of interest expenses to cover future excess interest payments. The Spanish and Portuguese legislations also allow for a 5-year EBITDA carryforward and Italy for an unlimited one. In the case of an unlimited EBITDA carryforward, denoted by EC_t , the EBITDA carryforward is in an arbitrary period t determined by

$$EC_{t+1} = \max(\beta \cdot EBITDA_{t+1} + EC_t - r_D \cdot D_t - IC_t, 0) \quad (20)$$

and consequently the tax deductible interest amount is calculated by

$$\Psi_{t+1} = \min(\beta \cdot EBITDA_{t+1} + EC_t, r_D \cdot D_t + IC_t). \quad (21)$$

In Germany¹¹ and Finland the earnings stripping rule contains a unique feature referred to as the escape clause which defines the circumstance where the earnings stripping is not applicable. This is the case when the book-equity-to-total-assets ratio of the financial statement of a group company is minimum

⁸The discussion of the introduction of an earnings stripping rule in Germany dates back to a resolution of the commission of important key points to the planned german tax reform 2008 on July 2, 2006.

⁹We assume that the considered firm does not earn any interest income.

¹⁰Art. 1 of the Growth Acceleration Act, December 22, 2009, BGBl 2009 I, 3950; applicable for fiscal years ending after December 31, 2009.

¹¹In Germany a book-equity-to-total-assets ratio of a group company less than 2% of the consolidated one is harmless.

as high as the corresponding ratio in the consolidated financial statements of the parent company. The different structures of the country-specific earnings stripping rules are depicted in Table 3.

In order to compare the effects on the tax shield of thin-capitalization rules and earnings stripping rules, we consider the same framework (parent-subsidiary) for our analysis of the earnings stripping rules¹². By following Appendix B, the tax shield under consideration of an earnings stripping rule can be determined by

$$TS_{t+1}^{ESR} = (I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub}) \cdot (1 - \tau_D) - (1 - \tau_P) \cdot [I_{t+1}^{par} + [(1 - y \cdot \tau_C) - (1 - \alpha) \cdot (1 - \tau_C)] \cdot I_{t+1}^{sub} - \tau_C \cdot \Psi_{t+1}^{par} - (1 - y \cdot \tau_C) \cdot \tau_C \cdot \Psi_{t+1}^{sub}], \quad (22)$$

where the tax deductible interest amount $\Psi_{t+1}^{(\cdot)}$ ¹³ for the parent and the subsidiary is given by

$$\Psi_{t+1}^{(\cdot)} = -\max(\beta \cdot EBITDA_{t+1}^{(\cdot)} + EC_t^{(\cdot)} - r_D \cdot D_t^{(\cdot)} - IC_t^{(\cdot)}, 0) + \beta \cdot EBITDA_{t+1}^{(\cdot)} + EC_t^{(\cdot)}, \quad (23)$$

the corresponding interest carryforward by

$$IC_{t+1}^{(\cdot)} = \max(r_D \cdot D_t^{(\cdot)} + IC_t^{(\cdot)} - \beta \cdot EBITDA_{t+1}^{(\cdot)} - EC_t^{(\cdot)}, 0). \quad (24)$$

For an adequate and detailed analysis of the integrated period specific tax savings given in equation (22), we disentangle this equation into its effect-relationships. The term $(1 - \tau_P) \cdot (1 - \alpha) \cdot (1 - \tau_C) \cdot I_t^{sub}$ determines the effect of the internal interest payments on parent-subsidiary debt financing on the individual level, subject to personal taxes on dividend payments. $(I_t^{par} + \alpha \cdot I_t^{sub}) \cdot (1 - \tau_D)$ represents the effect of interest payments on external debt financing after personal taxes on interest income. In general, the term $-(1 - \tau_P) \cdot [(\tau_C - y \cdot \tau_C) \cdot I_t^{sub} - (1 - y \cdot \tau_C) \cdot \tau_C \cdot \Psi_t^{sub}]$ maps the combined influence of the subsidiary's interest payments originating from internal parent-subsidiary debt financing. In particular, it represents the negative effect as after-tax income on the individual level considering personal taxes on dividend income, i.e. $-(1 - \tau_P) \cdot (\tau_C - y \cdot \tau_C) \cdot I_t^{sub}$, and the positive effect of the tax deductible interest payments of the subsidiary subject to the earnings stripping rule, i.e. $(1 - \tau_P) \cdot (1 - y \cdot \tau_C) \cdot \tau_C \cdot \Psi_t^{sub}$. It is important to note that due to a possible application of the earnings stripping rule, Ψ_t^{sub} might be bigger than I_t^{sub} . An equivalent interpretation without the additional taxation on intragroup dividends ($y \cdot \tau_C$) paid by the subsidiary to the parent holds for $-(1 - \tau_P) \cdot (I_t^{par} - \tau_C \cdot \Psi_t^{par})$. The important relation within the context of the tax deductible interest under the application of an earnings stripping rule is given by equation (23). Independent of the parent or the subsidiary's level, this equation can be interpreted in two parts: (1.) A basic and always tax deductible interest amount which is equal to $\beta \cdot EBITDA_t^{(\cdot)}$ (plus possible $EC_t^{(\cdot)}$) and (2.) a maximum function which determines the additional tax deductible interest in case the earnings stripping rule does not apply (plus possible $IC_t^{(\cdot)}$), i.e. $\beta \cdot EBITDA_t^{(\cdot)} > r_D \cdot D_{t-1}^{(\cdot)}$.

In this regard, we would like to turn attention on the difference between the tax benefits due to interest payments on the corporate level and the tax disadvantage due to interest income on the individual level in an arbitrary period t , i.e. $(\tau_C - \tau_D) \cdot I_t^{(\cdot)}$. This difference is positive as long as $\tau_C > \tau_D$. With an earnings stripping rule due to the limited tax deductibility of interest payments on the corporate level this difference is given by $\tau_C \cdot \Psi_{t+1}^{(\cdot)} - \tau_D \cdot I_{t+1}^{(\cdot)}$. For simplification purposes with $EC_t = 0$, the algebraic sign of this difference depends on whether the earnings stripping rule applies or not. In case the earnings stripping rule applies, this difference might become negative even for $\tau_C > \tau_D$ due to $I_{t+1}^{(\cdot)} > \Psi_{t+1}^{(\cdot)}$. In

¹²This is useful, because e.g. in Finland and Greece the non-deductibility criteria applies only for related parties.

¹³Note that we use the general expression (\cdot) within this context as superscript for indicating that the respective formulas are valid for the parent as well as the subsidiary.

Table 3
Country-specific Earnings Stripping Rules in the EU-15 Countries

Tax shield with earnings stripping rules								
	Certain percentage of EBITDA (β)	Exemption threshold ⁵	Interest carry-forward	Spare EBITDA carryforward	Type of company	Escape clause	Formula for $\varphi_{t+1}^{(c)}$	Formula for $IC_{t+1}^{(c)}$
FIN	30%	0.5	unlimited	-	related parties ⁴	yes	$\min(\beta \cdot EBITDA_{t+1}^{(c)} \cdot r_D \cdot D_t^{(c)} + IC_t^{(c)})$	$\max(r_D \cdot D_t^{(c)} + IC_t^{(c)} - \beta \cdot EBITDA_{t+1}^{(c)}, 0)$
GER	30%	3	unlimited	5 years	all companies	yes	$\min(\beta \cdot EBITDA_{t+1}^{(c)} + \sum_{i=t-4}^{t-1} EC_{n,i}^{(c)} - \sum_{i=t-4}^{t-1} EC_{n,i+1}^{(c)}, r_D \cdot D_t^{(c)} + IC_t^{(c)})$	$\max(r_D \cdot D_t^{(c)} + IC_t^{(c)} - \beta \cdot EBITDA_{t+1}^{(c)} - \sum_{i=t-4}^{t-1} EC_{n,i}^{(c)}, 0)$
GRE	60% ¹	5 ²	unlimited	-	group members	-	$\min(\beta \cdot EBITDA_{t+1}^{(c)}, r_D \cdot D_t^{(c)} + IC_t^{(c)})$	$\max(r_D \cdot D_t^{(c)} + IC_t^{(c)} - \beta \cdot EBITDA_{t+1}^{(c)}, 0)$
ITA	30%	-	unlimited	unlimited ³	all companies ⁴	-	$\min(\beta \cdot EBITDA_{t+1}^{(c)} + EC_t^{(c)}, r_D \cdot D_t^{(c)} + IC_t^{(c)})$	$\max(r_D \cdot D_t^{(c)} + IC_t^{(c)} - \beta \cdot EBITDA_{t+1}^{(c)} - EC_t^{(c)}, 0)$
POR	60% ¹	3	5 years	5 years	all companies	-	$\min(\beta \cdot EBITDA_{t+1}^{(c)} + \sum_{i=t-4}^{t-1} EC_{n,i}^{(c)} - \sum_{i=t-4}^{t-1} EC_{n,i+1}^{(c)}, r_D \cdot D_t^{(c)} + \sum_{i=t-4}^{t-1} IC_{n,i}^{(c)} - \sum_{i=t-4}^{t-1} IC_{n,i+1}^{(c)})$	
ESP	30%	1	18 years	5 years	all companies	-	$\min(\beta \cdot EBITDA_{t+1}^{(c)} + \sum_{i=t-4}^{t-1} EC_{n,i}^{(c)} - \sum_{i=t-4}^{t-1} EC_{n,i+1}^{(c)}, r_D \cdot D_t^{(c)} + \sum_{i=t-4}^{t-1} IC_{n,i}^{(c)} - \sum_{i=t-4}^{t-1} IC_{n,i+1}^{(c)})$	
GER POR ESP	$EC_{n,t}^{(c)} = \begin{cases} \max(\beta \cdot EBITDA_n^{(c)} - r_D \cdot D_{t-1}^{(c)}, 0), \\ \max(\sum_{i=t-4}^{t-1} EC_{n,i}^{(c)} - \sum_{i=t-4}^{t-1} EC_{n,i-1}^{(c)} - \max(r_D \cdot D_{t-1}^{(c)} - \beta \cdot EBITDA_n^{(c)}, 0), 0), \\ 0, \end{cases}$				$\text{for } n = t$ $\text{for } t - 5 \leq n \leq t - 1$ $\text{for } t - 5 > n$	ITA	$EC_t^{(c)} = \max(\beta \cdot EBITDA_t^{(c)} + EC_{t-1}^{(c)} - r_D \cdot D_{t-1}^{(c)} - IC_{t-1}^{(c)}, 0)$	
POR	$IC_{n,t}^{(c)} = \begin{cases} \max(\beta \cdot EBITDA_n^{(c)} + \sum_{i=t-5}^{t-1} EC_{n,i}^{(c)} - r_D \cdot D_{t-1}^{(c)}, 0), \\ \max(\sum_{i=t-4}^{t-1} IC_{n,i}^{(c)} - \sum_{i=t-4}^{t-1} IC_{n,i-1}^{(c)} - \max(\beta \cdot EBITDA_t^{(c)} + \sum_{i=t-5}^{t-1} EC_{n,i}^{(c)} - r_D \cdot D_{t-1}^{(c)}, 0), 0), \\ 0, \end{cases}$				$\text{for } n = t$ $\text{for } t - 5 \leq n \leq t - 1$ $\text{for } t - 5 > n$			
ESP	$IC_{n,t}^{(c)} = \begin{cases} \max(\beta \cdot EBITDA_n^{(c)} + \sum_{i=t-5}^{t-1} EC_{n,i}^{(c)} - r_D \cdot D_{t-1}^{(c)}, 0), \\ \max(\sum_{i=t-4}^{t-1} IC_{n,i}^{(c)} - \sum_{i=t-4}^{t-1} IC_{n,i-1}^{(c)} - \max(\beta \cdot EBITDA_t^{(c)} + \sum_{i=t-5}^{t-1} EC_{n,i}^{(c)} - r_D \cdot D_{t-1}^{(c)}, 0), 0), \\ 0, \end{cases}$				$\text{for } n = t$ $\text{for } t - 5 \leq n \leq t - 1$ $\text{for } t - 5 > n$			

¹ In 2014, reducing each year by 10% until 2017 with 30%.

² From 2014-2015, € 3 million from 2016.

³ Excess interest expenses of a group company may be offset with spare EBITDA capacity of another group company.

⁴ Further exemptions for financial, insurance and pension institutions and partially for associated group companies and mutual real estate and housing companies.

⁵ In € million.

case the earnings stripping rule does not apply, we have two possible scenarios depending on $IC_t^{(\cdot)}$. For $IC_t^{(\cdot)} = 0$ and $I_{t+1}^{(\cdot)} = \Psi_{t+1}^{(\cdot)}$, the algebraic sign of the difference depends again on the relation of τ_C and τ_D . For a positive $IC_t^{(\cdot)}$ we have $\Psi_{t+1}^{(\cdot)} > I_{t+1}^{(\cdot)}$ and the difference might become positive.

With the period specific tax savings subject to a possible application of an earnings stripping rule, the tax shield value under the risk-neutral probability measure \mathbb{Q} contingent on the available information in t is determined by

$$VTS_t^{ESR} = \sum_{s=t+1}^T \frac{E_t^{\mathbb{Q}}[TS_s^{ESR}]}{(1+r_f)^{s-t}} \quad (25)$$

By substituting the equations (22) and (23) for the parent and the subsidiary into (25) and rearranging we get

$$\begin{aligned} VTS_t^{ESR} = & \sum_{s=t+1}^T \left(\frac{E_t^{\mathbb{Q}} \left[\left(I_s^{par} + \alpha \cdot I_s^{sub} \right) \cdot (1 - \tau_D) - (1 - \tau_P) \cdot \left[I_s^{par} + [(1 - y \cdot \tau_C) - (1 - \alpha) \cdot (1 - \tau_C)] \cdot I_s^{sub} \right] \right]}{(1 + r_f)^{s-t}} \right. \\ & + (1 - \tau_P) \cdot \tau_C \cdot \left(\frac{E_t^{\mathbb{Q}} \left[\beta \cdot EBITDA_s^{par} + EC_{s-1}^{par} - \max(\beta \cdot EBITDA_s^{par} + EC_{s-1}^{par} - I_s^{par} - IC_{s-1}^{par}, 0) \right]}{(1 + r_f)^{s-t}} \right. \\ & \left. \left. + (1 - y \cdot \tau_C) \cdot \frac{E_t^{\mathbb{Q}} \left[\beta \cdot EBITDA_s^{sub} + EC_{s-1}^{sub} - \max(\beta \cdot EBITDA_s^{sub} + EC_{s-1}^{sub} - I_s^{sub} - IC_{s-1}^{sub}, 0) \right]}{(1 + r_f)^{s-t}} \right] \right). \end{aligned} \quad (26)$$

Basically, equation (26) consists of three fractions constituting the tax shield value. We start by discussing the value increments of the first fraction. The increment $(I_s^{par} + \alpha \cdot I_s^{sub}) \cdot (1 - \tau_D)$ maps the additional tax burden on the individual level after taxes on interest income caused by the interest payments of the overall group. With $\alpha = 0$, i.e. the subsidiary is fully financed by parent-subsidiary financing, this term only represents the additional tax burden caused by debt financing of the parent. $(1 - \tau_P) \cdot (1 - \alpha) \cdot (1 - \tau_C) \cdot I_s^{sub}$ determines the effect of tax payments on dividends of increased dividends payments by the parent to the individual investors which originate from interests paid by the subsidiary to the parent. The term $-(1 - \tau_P) \cdot [I_s^{par} + (1 - y \cdot \tau_C) \cdot I_s^{sub}]$ represents the combined effect of smaller tax payments on dividends on the individual level due to the fact that interest payments reduce dividend payments. The second and third fraction represent the value of the tax deductible interest payments for the parent and the subsidiary after personal taxes on dividend payments. In any case, whether the earnings stripping rule applies or not, the added fractions determine the present value of all future tax savings $\beta \cdot EBITDA_s$ and possible EC_{s-1} , $\forall s > t$, for the overall group.

4.2. Numerical example: earnings stripping rules

In this subsection, we outline a numerical example for an exemplifying earnings stripping rule. In order to compare the influence of different interest limitation rules, we rewrite equation (26) by

$$\begin{aligned} VTS_t^{ESR} &= \sum_{s=t+1}^T \frac{E_t^Q[TS_s^N]}{(1+r_f)^{s-t}} - \sum_{s=t+1}^T \frac{E_t^Q[TS_s^N - TS_s^{ESR}]}{(1+r_f)^{s-t}} \\ &= \sum_{s=t+1}^T \frac{E_t^Q[TS_s^N]}{(1+r_f)^{s-t}} - \sum_{s=t+1}^T \frac{E_t^Q \left[(1-\tau_P) \cdot \tau_C \cdot \left[I_s^{par} - \Psi_s^{par} + (1-y \cdot \tau_C) \cdot (I_s^{sub} - \Psi_s^{sub}) \right] \right]}{(1+r_f)^{s-t}}, \end{aligned} \quad (27)$$

where the term $(1-\tau_P) \cdot \tau_C \cdot (1-y \cdot \tau_C) \cdot (I_{t+1}^s - \Psi_{t+1}^s)$ expresses the negative influence of non-deductible interest expenses or in case that $\Psi_{t+1}^s > I_{t+1}^s$ the positive influence of an earnings stripping rule on the tax shield.

We use the assumptions and basis parameters as described in section 3.2. Since the application of the earnings stripping rule depends on the value of the EBITDA, we need to find a functional relation between the FCF_t^U and $EBITDA_t$ for our subsequent analysis. The earnings before interest and tax (EBIT) may be defined in proportion to the EBITDA

$$EBIT_t = \delta \cdot EBITDA_t. \quad (28)$$

The factor δ , with $\delta \in [0, 1]$ ¹⁴, maps the effect of depreciation and amortization $Depr_t = (1-\delta) \cdot EBITDA_t$. The definition of FCF_t^U under consideration of corporate taxes τ_C , depreciation and amortization and investments Inv_t is given by

$$FCF_t^U = (1-\tau_C) \cdot EBIT_t + (1-\delta) \cdot EBITDA_t - Inv_t \quad (29)$$

For simplifying the following analysis we set $Inv_t = 0$. The definition of $EBITDA_t$ in relation to FCF_t^U is given by

$$\begin{aligned} FCF_t^U &= (1-\tau_C) \cdot \delta \cdot EBITDA_t + (1-\delta) \cdot EBITDA_t \\ FCF_t^U &= (\delta - \delta \cdot \tau_C + 1 - \delta) \cdot EBITDA_t \\ EBITDA_t &= \frac{FCF_t^U}{(1-\delta \cdot \tau_C)} \end{aligned} \quad (30)$$

If we set $\delta = 1$ ¹⁵, we get $EBITDA_t = \frac{FCF_t^U}{(1-\tau_C)}$.

Figure 6 gives a detailed overview on the state-dependent evolvement of the parameters and the tax shield value. The tax shield of the individual investor without any interest limitations according to equation (7) generated by a subsidiaries debt of $D^{sub} = 700$ amounts to $TS_t^N = 5.583$ (in a single period $TS_t^N = 0.319$). By considering the application of an earnings stripping rule according to equation (27) the overall tax shield amounts to $VTS_t^{ESR} = 5.583 - 2.766 = 2.817$. We separately show the state dependent values of the EBITDA and the interest carryforward in Figure 7.

¹⁴See for example Eberl (2009), p. 269.

¹⁵It is important to note that we implicitly assume that the firm always reinvests the depreciation.

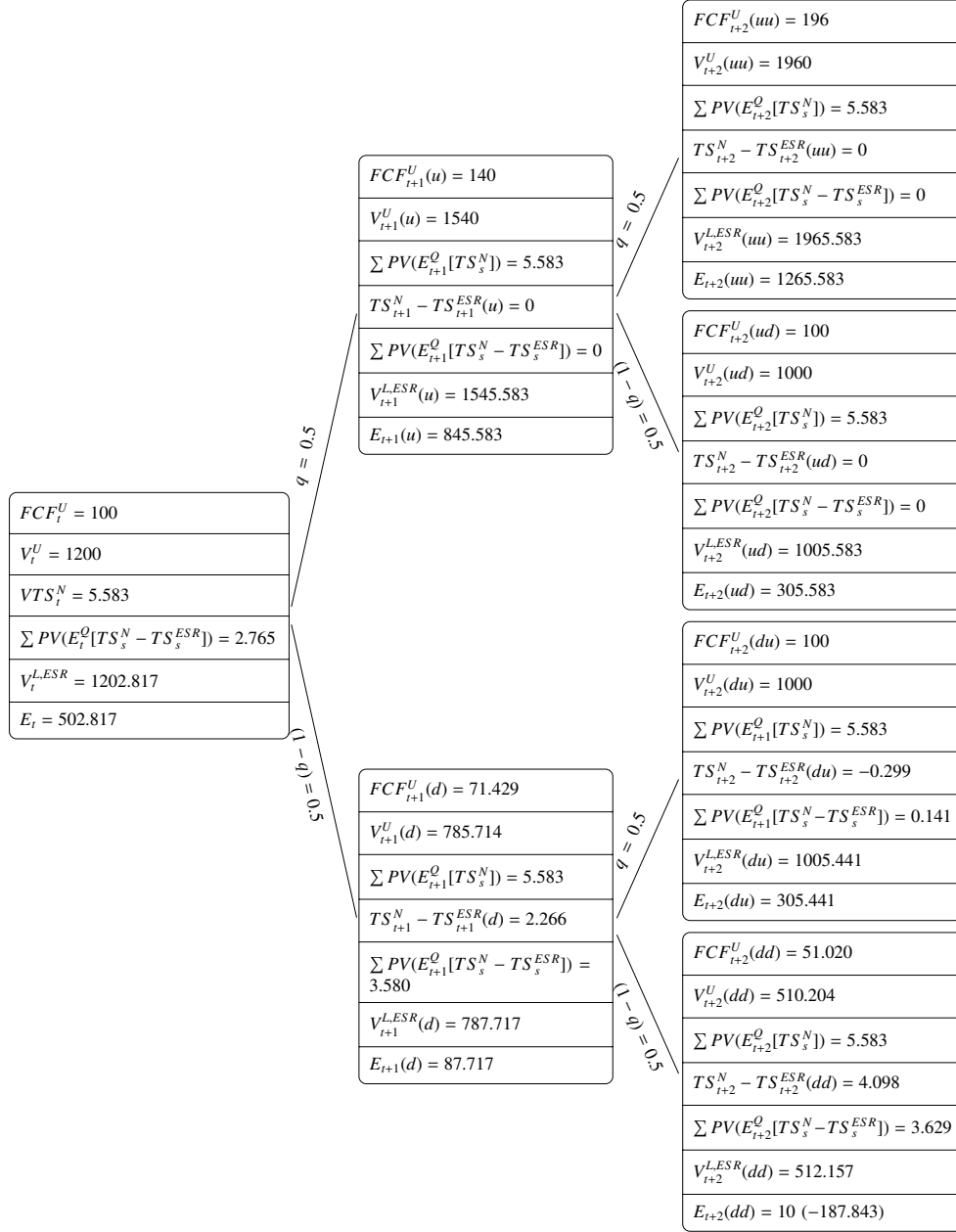


Fig. 6
Numerical Example for an Earnings Stripping Rule.

Throughout this numerical example we use the parameters as given in section 4.2. As explained in figure 4 and 5 we have abstained from depicting periods with $s > t + 2$ and from period $t + 4$ onwards we have assumed for ease of calculation a perpetual tax shield of 5.583. The different state-dependent quantities are denoted by the respective up- or down-movements. We show in case of indebtedness the implied negative equity value. Obviously, in case of a limited liability firm the equity value is then zero; concerning our assumptions it is injected up to 10.

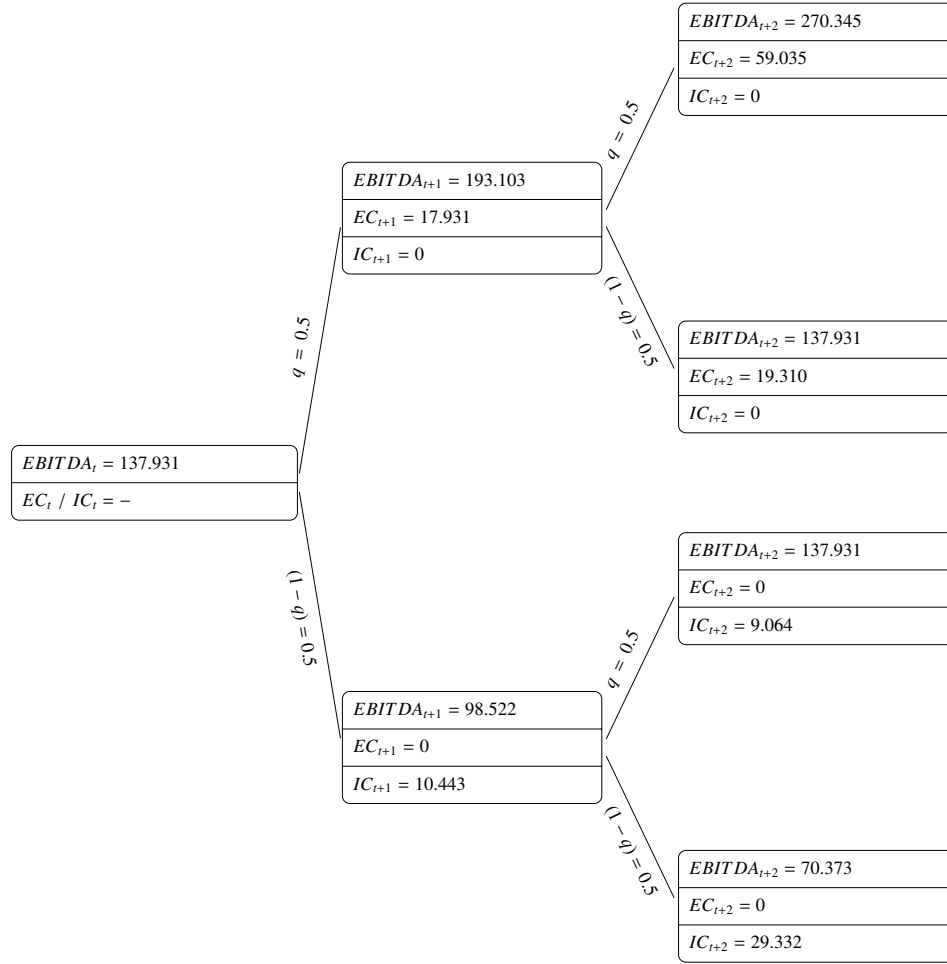


Fig. 7
EBITDA_t, EC_t, IC_t for the described States.

In this binomial lattice we use the parameters as given in section 4.2. It demonstrates the explicit EBITDA_t calculated by $\frac{FCF_t^U}{(1-\tau_C)}$, IC_t calculated by equation (18) and EC_t by equation (20) in the specific period. The single parameters are not recombining with each other. At time *t* interest expenses do not exist and therefore there is no EC_t / IC_t. The results of all calculations are rounded to three digits.

In order to point out the effect of the earnings stripping rule we discuss in detail these exemplifying states: u and dd . In the u -state all interest expenses $r_D \cdot D_t^{sub} = 0.05714 \cdot 700 = 40$ may be deducted as the deductible interest amount from representation (23) is

$$\Psi_{t+1}^{sub} = -\max(0.3 \cdot 193.103 - 40, 0) + 57.931 = -\max(17.931, 0) + 57.931 = 40. \quad (31)$$

Thus, no interest carryforward (18) exists

$$IC_{t+1}^{sub} = \max(0.05714 \cdot 700 - 0.3 \cdot 193.103, 0) = \max(-17.931, 0) = 0 \quad (32)$$

and the spare EBITDA (20)

$$EC_{t+1}^{sub} = \max(57.931 - 40, 0) = \max(17.931, 0) = 17.931 \quad (33)$$

may be carried forward in future periods. The tax shield amounts to $TS_{t+1}^{ESR} = 0.319$

The dd -state shows an application of the earnings stripping rule in which only a fraction of the interest expense is tax-deductible

$$\begin{aligned} \Psi_{t+2}^{sub} &= -\max(0.3 \cdot 70.373 + 0 - 40 - 10.443, 0) + 21.112 \\ &= -\max(-29.332, 0) + 21.112 = 21.112. \end{aligned} \quad (34)$$

Since the non deductible amount of the interest payments can be carried forward the interest carryforward amounts to

$$IC_{t+2}^{sub} = \max(40 + 10.443 - 21.112, 0) = \max(29.332, 0) = 29.332. \quad (35)$$

Consequently, there is no spare EBITDA to carry forward in future periods

$$EC_{t+2}^{sub} = \max(21.112 - 50.443, 0) = -\max(-29.332, 0) = 0. \quad (36)$$

The value of equation (22) is negative

$$\begin{aligned} TS_{t+2}^{ESR} &= 0 \cdot 40 \cdot (1 - 0.2) - (1 - 0.2) \cdot [(1 - 0.05 \cdot 0.275) - (1 - 0) \cdot (1 - 0.275)] \cdot 40 \\ &\quad - (1 - 0.05 \cdot 0.275) \cdot 0.275 \cdot 21.112 \\ &= -(1 - 0.2) \cdot (10.450 - 5.726) = -3.779. \end{aligned} \quad (37)$$

The impact of the earnings stripping rule on the tax shield amounts to $TS_{t+2}^N - TS_{t+2}^{ESR} = 4.098$ and to demonstrate the tax shield with equation (27), it amounts (equivalently to equation (22)) to $TS_{t+1}^{ESR} = 0.319 - 4.098 = -3.779$.

As aforementioned with an earnings stripping rule, there might be a positive influence on the tax shield in a single period due to an interest carryforward. Such a scenario can be found in the du -state. Here the total amount of interest expenses can be deducted and in addition a fraction of the IC_{t+1}^{sub} is tax deductible:

$$\begin{aligned} \Psi_{t+2}^{sub} &= -\max(0.3 \cdot 137.931 + 0 - 40 - 10.443, 0) + 41.379 \\ &= -\max(-9.064, 0) + 41.379 = 41.379. \end{aligned} \quad (38)$$

Hence, the state-dependent tax savings $TS_{t+2}^{ESR} = 0.614$ rise above the value TS_{t+2}^N by 0.299. The corresponding values for the carryforwards are $EC_{t+2}^{sub} = 0$ and $IC_{t+2}^{sub} = 9.064$.

5. Implications for the Valuation of Tax Shields

Within the subsequent section, we aim at comparing the above discussed thin-capitalization and earnings stripping rules by determining their respective impact on the tax shield value. In order to illustrate the effect, we relate the values subject to the limited tax deductibility of interest to the tax shield value without any limitation (VTS_t^N). For this numerical comparison we use the parameters from above, which are summarized to: $FCF_t^U = 100$, $u = 1.4$, $q = 0.5$, $r_f = 5.714\%$, $\alpha = 0$, $\frac{D^{TC}}{E^{TC}} = 4$, $\tau_C = 27.5\%$, $\tau_P = 20\%$, $\tau_D = 20\%$, $j = 1$, $h = 1$, $\beta = 0.3$, and $\delta = 1$, while we vary the total amount of debt from 200 to 1,200.

In order to quantify the difference between the tax shield values with and without the limiting rules, we calculate the percentage differences for the tax shield and the levered firm value. We define the percentage difference of the tax shield values with and without limitation by

$$TS\text{-difference} = \frac{VTS_t^N - VTS_t^{(.)}}{VTS_t^N} = \frac{\Delta TS}{VTS_t^N} (\%), \quad (39)$$

where the superscript $(.)$ is in this respect the placeholder for the respective tax rule and the expression ΔTS indicates the accumulated present values of the respective tax shield difference. A positive value indicates the relative loss in tax shield value which results from a possible application of the discussed rules. Additionally, this reveals for values above 100% that due to respective tax treatment, the full tax savings are lost and that debt financing has an overall negative value contribution. An equivalent analysis can be conducted by determining the percentage difference for the levered firm values, i.e.

$$V^L\text{-difference} = \frac{V_t^{L,N} - V_t^{L,(.)}}{V_t^{L,N}} = \frac{\Delta V^L}{V_t^{L,N}} (\%). \quad (40)$$

Table 4 provides an aggregated view on the effect of the thin-capitalization and earnings stripping rules by showing the tax shield values (VTS_t^N) and the percentage share of the standard tax shield without any limitation to the levered firm value ($\frac{VTS_t^N}{V_t^{L,N}}$), the direct impact of the respective tax rule (ΔTS) and the above defined relative value differences. For completeness we depict the percentage share of the final tax shield including interest limitation rules to the levered firm value without limitation rules ($\frac{VTS_t^{(.)}}{V_t^{L,N}}$) as well as the implied levered firm ($V_t^{L,(.)}$) and equity values E_t as well.

First, let us regard the tax shield without any limitations. In relation to the levered firm value the percentage value contribution shows values between 0.13% and 0.8% with increasing debt values. Needless to say that this small relative value contribution results from the fact that the tax shield is measured on the parental level and therefore considers the double taxation effect of passing the subsidiary's dividends on to the parent. Nevertheless, speaking in absolute terms, for a firm with a value of €5 billion the tax shield would amount to €40 million which is a non-neglectable value.

Second, we examine the impact of the interest limitation rules on the tax shield (and on the levered firm value). We observe that the earnings stripping rule is first applied at a debt level of 400, while the two cases of the thin-capitalization rule are applicable at a debt level of 500. The smallest relative difference on the tax shield value can be recognized for the rec-case at a debt level of 500 with 1.074% (0.004%), while for the ESR-case the relative impact amounts to 2.032% (0.005%) already at a debt level of 400. The highest possible discount can be surveyed for the nor-case at a debt level of 1,200 with 304.808% (2.412%). This shows that the impact of the limited interest deductibility exceeds the tax shield without limitation and therefore indicates a tax advantage of equity financing. This effect can already be recognized at a debt level of 900 for the nor-case. The same effect can be observed for the ESR-case at a debt level of 1,000.

Table 4
Tax Shields and Firm Values depending on the Debt Ratio for all Interest Deduction Cases

This table shows the impact of the thin-capitalization and earnings stripping rules on the tax shield value ($V_t^{L,N}$) and the levered firm value ($V_t^{L,N}$) without any limitation of the tax deductibility of interest payments. The parameters of the numerical example are set to $FCF_t = 100$, $u = 1.4$, $q = 0.5$, $r_f = 5.714\%$, $\alpha = 0$, $\frac{D}{E} = 27.5\%$, $\tau_P = 20\%$, $\tau_D = 20\%$, $j = 1$, $h = 1$, $\beta = 0.3$, and $\delta = 1$, while we vary the total amount of debt from 200 to 1,200. ATS shows the respective values for the considered tax rules, $\frac{V_t^{L,N}}{V_t^{L,N}}$ the value contribution of the tax shield without limitation on the overall firm value, $\frac{V_t^{L,N} - V_t^{L,(.)}}{V_t^{L,N}}$ and $\frac{V_t^{L,N} - V_t^{L,(.)}}{V_t^{L,N}}$ (%) as discussed in section 5 within the equation (39) and (40).

D (debt ratio (%))	$V_t^{L,N}$	$\frac{V_t^{L,N}}{V_t^{L,N}}$ (%)	case	$ATS = \begin{cases} \sum PV(E_t^Q) & \text{if } (max(\cdot)) \\ \sum PV(E_t^Q) & \text{if } (TS_N - TS_{ESR}) \end{cases}$	$\frac{V_t^{L,N} - V_t^{L,(.)}}{V_t^{L,N}}$ (%)	$\frac{V_t^{L,N} - V_t^{L,(.)}}{V_t^{L,N}}$ (%)	$\frac{V_t^{L,N} - ATS}{V_t^{L,N}}$ (%)	$V_t^{L,(.)}$	$E_t^{(.)}$
200 (16.66%)	1.595	0.133	rec/nor/ESR	0.000	0.000	0.000	0.133	1201.595	1001.595
300 (25.00%)	2.393	0.199	rec/nor/ESR	0.000	0.000	0.000	0.199	1202.393	902.393
400 (33.33%)	3.190	0.265	rec/nor/ESR	0.000	0.000	0.000	0.265	1203.190	803.190
500 (41.66%)	3.988	0.331	rec/nor/ESR	0.065	2.032	0.005	0.260	1203.125	803.125
600 (50.00%)	4.785	0.397	rec/nor/ESR	0.601	15.081	0.050	0.281	1203.386	703.386
700 (58.33%)	5.583	0.463	rec/nor/ESR	1.469	30.706	0.122	0.276	1203.316	603.316
800 (66.66%)	6.380	0.529	rec/nor/ESR	2.765	49.531	0.229	0.234	1202.817	502.817
900 (75.00%)	7.178	0.595	rec/nor/ESR	4.417	69.238	0.366	0.163	1201.963	401.963
1000 (83.33%)	7.975	0.660	rec/nor/ESR	6.210	97.333	0.515	0.014	1200.170	300.691
1200 (100.00%)	9.570	0.792	rec/nor/ESR	14.443	150.916	1.194	-0.408	1195.127	10.000

Finally, we compare the different influences of the respective cases. The rec-case shows the smallest relative impact with values up to 11.137% (0.088%). As a direct result of the reclassification of non-deductible interest as dividend payments, this implies that the reclassified part of the interest payments is equally taxed as if the respective proportion of debt would have been equity financed (or financed by debt from a third party). In contrast to this, in the nor- and ESR-case, the non-deductible part of the interest payments remains taxed as interest income which is taxed at a higher effective tax rate (due to a higher tax base than dividend income). As already touched above, the earnings stripping rule applies at a lower debt level (400) than thin-capitalization rules (500), while the nor-case already causes the highest reduction on the tax shield at a debt level of 700.

Overall, we find that thin-capitalization and earnings stripping rules have an important impact on the tax shield and in turn on the levered firm value. Evidently, with increasing leverage the value discount on the tax shield increases as well. In our example the relative impact on the tax shield varies between 1 and 305%. Moreover, due to the different definitions of the earnings stripping and thin-capitalization rules, any comparison among the rules has to be based on the major parameters, i.e. the EBITDA and the debt-to-equity ratio respectively. Summarizing, for firms with high leverage it is important to consider the effects of limited interest tax deductibility.

6. Conclusions

This paper analyzes the valuation of tax shields when specific tax rules limit the tax deductibility of interest payments on debt. Focussing on the specific tax codes of the EU-15 countries, we aim at setting up a theoretical framework that enables us to compare thin-capitalization rules limiting the tax deductibility of interest payments originating from intragroup financing and earnings stripping rules. We are able to provide a tractable tax shield and firm valuation model that includes the possibility of a limitation of the tax deductibility of interest implied by the regarded tax codes.

We show how the necessary adjustments can be incorporated in a tax shield valuation framework considering personal taxes. Thereby, we provide a general framework how specific tax codes limiting the tax deductibility of interest can be integrated and show the respective valuation equation for the EU-15 countries. The 'new' tax shield representation reveals that the present value of the payoff-function considering a possible limitation of interest tax deductibility is subtracted from the standard tax shield. Depending on the tax code this payoff-function and in turn the overall tax shield shows a path-dependent feature.

Furthermore, in our analysis we highlight that for firms with a low leverage, there is no or a rather small impact of thin-capitalization and earnings stripping rules on the tax shield value. Nevertheless, for firms with a high leverage, the impact of these rules results in an important and non-neglectable discount in the tax shield value. In some of the cases we observe that the impact has as consequence that the benefits from debt financing could turn into a disadvantage.

The results obtained are important not only for company valuation in business practice. Additionally, the paper contributes to the academic tax shield discussion and provides valuation equations that could serve as a measure for the benefits of debt financing that play an important role in other research fields such as capital structure. We believe that more research within the field of tax shield valuation is necessary to fully understand its impact on the overall firm.

Appendix A Derivation of Tax Shield Equations and Explanations for Thin-Capitalization Rules

A.1 Tax shield equations for general parent-subsidiary financing without limitation rules

The after-tax income of an unlevered subsidiary is given by

$$FCF_{t+1}^{U,sub} = (1 - \tau_C) \cdot EBIT_{t+1}^{sub} + Depr_{t+1} - Inv_{t+1},$$

where $Depr_{t+1}$ denotes the depreciation and Inv_{t+1} the investments. For simplicity purposes we set $Depr_{t+1} = Inv_{t+1}$. Thereby, the unlevered free cash flows are determined by

$$FCF_{t+1}^{U,sub} = (1 - \tau_C) \cdot EBIT_{t+1}^{sub}.$$

By distributing 100% of the subsidiary's after tax income to the parent and assuming that only a fraction y of the distributed dividend is taxable at the parent level, the after tax income of the parent is given by

$$\begin{aligned} FCF_{t+1}^{U,par} &= (1 - \tau_C) \cdot EBIT_{t+1}^{par} + (1 - \tau_C) \cdot (1 - y \cdot \tau_C) \cdot EBIT_{t+1}^{sub} \\ &= (1 - \tau_C) \cdot [EBIT_{t+1}^{par} + (1 - y \cdot \tau_C) \cdot EBIT_{t+1}^{sub}]. \end{aligned}$$

Hence, the after tax income of an individual investor of an unlevered firm is determined by

$$FCF_{t+1}^U = (1 - \tau_C) \cdot (1 - \tau_P) \cdot [EBIT_{t+1}^{par} + (1 - y \cdot \tau_C) \cdot EBIT_{t+1}^{sub}].$$

The after-tax income of a levered subsidiary without limitations on the tax deductibility of interest payments is given by

$$FCF_{t+1}^{L,sub} = (1 - \tau_C) \cdot (EBIT_{t+1}^{sub} - I_{t+1}^{sub}).$$

By distributing 100% of the subsidiary's after-tax income to the parent and assuming that a proportion y of the distributed dividend is taxable at the parent level, the after tax income of the parent is calculated by

$$\begin{aligned} FCF_{t+1}^{L,par} &= (1 - \tau_C) \cdot [EBIT_{t+1}^{par} - I_{t+1}^{par} + (1 - \alpha) \cdot I_{t+1}^{sub}] + (1 - \tau_C) \cdot (1 - y \cdot \tau_C) \cdot (EBIT_{t+1}^{sub} - I_{t+1}^{sub}) \\ &= (1 - \tau_C) \cdot (EBIT_{t+1}^{par} - I_{t+1}^{par} + I_{t+1}^{sub} - \alpha \cdot I_{t+1}^{sub} + EBIT_{t+1}^{sub} - I_{t+1}^{sub} - y \cdot \tau_C \cdot EBIT_{t+1}^{sub} + y \cdot \tau_C \cdot I_{t+1}^{sub}) \\ &= (1 - \tau_C) \cdot [EBIT_{t+1}^{par} - I_{t+1}^{par} + (1 - y \cdot \tau_C) \cdot EBIT_{t+1}^{sub} - (\alpha - y \cdot \tau_C) \cdot I_{t+1}^{sub}]. \end{aligned}$$

The after tax income of an individual investor of a levered firm is given by

$$\begin{aligned} FCF_{t+1}^L &= (I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub}) \cdot (1 - \tau_D) \\ &\quad + (1 - \tau_C) \cdot (1 - \tau_P) \cdot [EBIT_{t+1}^{par} - I_{t+1}^{par} + (1 - y \cdot \tau_C) \cdot EBIT_{t+1}^{sub} - (\alpha - y \cdot \tau_C) \cdot I_{t+1}^{sub}]. \end{aligned}$$

In any case, the period specific tax savings are determined as difference between the levered and

unlevered free cash flows

$$\begin{aligned}
TS_{t+1}^N &= FCF_{t+1}^L - FCF_{t+1}^U \\
&= (I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub}) \cdot (1 - \tau_D) + (1 - \tau_C) \cdot (1 - \tau_P) \cdot [EBIT_{t+1}^{par} - I_{t+1}^{par} + (1 - y \cdot \tau_C) \cdot EBIT_{t+1}^{sub} \\
&\quad - (\alpha - y \cdot \tau_C) \cdot I_{t+1}^{sub}] - (1 - \tau_C) \cdot (1 - \tau_P) \cdot [EBIT_{t+1}^{par} + (1 - y \cdot \tau_C) \cdot EBIT_{t+1}^{sub}] \\
&= (I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub}) \cdot (1 - \tau_D) + (1 - \tau_C) \cdot (1 - \tau_P) \cdot [-I_{t+1}^{par} - (\alpha - y \cdot \tau_C) \cdot I_{t+1}^{sub}] \\
&= (I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub}) \cdot (1 - \tau_D) - (1 - \tau_C) \cdot (1 - \tau_P) \cdot [I_{t+1}^{par} + (\alpha - y \cdot \tau_C) \cdot I_{t+1}^{sub}].
\end{aligned}$$

A.2 Tax shield equations for the rec-case

In case that a thin capitalization rule applies, the subsidiary's after-tax income including the non-deductible interest expense $I_{t+1}^{sub} - H_{t+1}$ is defined as

$$\begin{aligned}
FCF_{t+1}^{L,TC,sub} &= (1 - \tau_C) \cdot (EBIT_{t+1}^{sub} - \alpha \cdot I_{t+1}^{sub} - H_{t+1}) - [(1 - \alpha) \cdot I_{t+1}^{sub} - H_{t+1}] \\
&= EBIT_{t+1}^{sub} - \alpha \cdot I_{t+1}^{sub} - H_{t+1} - \tau_C \cdot EBIT_{t+1}^{sub} + \tau_C \cdot \alpha \cdot I_{t+1}^{sub} + \tau_C \cdot H_{t+1} - I_{t+1}^{sub} + \alpha \cdot I_{t+1}^{sub} + H_{t+1} \\
&= (1 - \tau_C) \cdot EBIT_{t+1}^{sub} - I_{t+1}^{sub} + \tau_C \cdot (\alpha \cdot I_{t+1}^{sub} + H_{t+1}).
\end{aligned}$$

The after-tax income of the parent including the subsidiary's dividend income in case of reclassification (rec) is given by

$$\begin{aligned}
FCF_{t+1}^{L,rec,par} &= (1 - \tau_C) \cdot (EBIT_{t+1}^{par} - I_{t+1}^{par} + H_{t+1}) \\
&\quad + (1 - y \cdot \tau_C) \cdot [(1 - \tau_C) \cdot EBIT_{t+1}^{sub} - I_{t+1}^{sub} + \tau_C \cdot (\alpha \cdot I_{t+1}^{sub} + H_{t+1}) + (1 - \alpha) \cdot I_{t+1}^{sub} - H_{t+1}] \\
&= (1 - \tau_C) \cdot (EBIT_{t+1}^{par} - I_{t+1}^{par} + H_{t+1}) \\
&\quad + (1 - y \cdot \tau_C) \cdot [(1 - \tau_C) \cdot EBIT_{t+1}^{sub} - I_{t+1}^{sub} + \tau_C \cdot \alpha \cdot I_{t+1}^{sub} + \tau_C \cdot H_{t+1} + I_{t+1}^{sub} - \alpha \cdot I_{t+1}^{sub} - H_{t+1}] \\
&= (1 - \tau_C) \cdot (EBIT_{t+1}^{par} - I_{t+1}^{par} + H_{t+1}) + (1 - y \cdot \tau_C) \cdot [(1 - \tau_C) \cdot EBIT_{t+1}^{sub} - (1 - \tau_C) \cdot \alpha \cdot I_{t+1}^{sub} - (1 - \tau_C) \cdot H_{t+1}] \\
&= (1 - \tau_C) \cdot (EBIT_{t+1}^{par} - I_{t+1}^{par} + H_{t+1}) + (1 - \tau_C) \cdot (1 - y \cdot \tau_C) \cdot (EBIT_{t+1}^{sub} - \alpha \cdot I_{t+1}^{sub} - H_{t+1}) \\
&= (1 - \tau_C) \cdot [EBIT_{t+1}^{par} - I_{t+1}^{par} + H_{t+1} + (1 - y \cdot \tau_C) \cdot (EBIT_{t+1}^{sub} - \alpha \cdot I_{t+1}^{sub} - H_{t+1})].
\end{aligned}$$

This in turn implies that the after-tax income of an individual investor of a levered firm is determined by

$$\begin{aligned}
FCF_{t+1}^{L,rec} &= (I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub}) \cdot (1 - \tau_D) + (1 - \tau_C) \cdot (1 - \tau_P) \cdot [EBIT_{t+1}^{par} - I_{t+1}^{par} + H_{t+1} \\
&\quad + (1 - y \cdot \tau_C) \cdot (EBIT_{t+1}^{sub} - \alpha \cdot I_{t+1}^{sub} - H_{t+1})].
\end{aligned}$$

The tax shield under consideration of a thin-capitalization rule in the rec-case is calculated as

$$\begin{aligned}
TS_{t+1}^{rec} &= FCF_{t+1}^{L,rec} - FCF_{t+1}^U \\
&= \left(I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub} \right) \cdot (1 - \tau_D) + (1 - \tau_C) \cdot (1 - \tau_P) \cdot \left[EBIT_{t+1}^{par} - I_{t+1}^{par} + H_{t+1} \right. \\
&\quad \left. + (1 - y \cdot \tau_C) \cdot (EBIT_{t+1}^{sub} - \alpha \cdot I_{t+1}^{sub} - H_{t+1}) \right] \\
&\quad - (1 - \tau_C) \cdot (1 - \tau_P) \cdot \left[EBIT_{t+1}^{par} + (1 - y \cdot \tau_C) \cdot EBIT_{t+1}^{sub} \right] \\
&= \left(I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub} \right) \cdot (1 - \tau_D) + (1 - \tau_C) \cdot (1 - \tau_P) \cdot \left[-I_{t+1}^{par} + H_{t+1} - (1 - y \cdot \tau_C) \cdot (\alpha \cdot I_{t+1}^{sub} + H_{t+1}) \right] \\
&= \left(I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub} \right) \cdot (1 - \tau_D) - (1 - \tau_C) \cdot (1 - \tau_P) \cdot \left[I_{t+1}^{par} + (1 - y \cdot \tau_C) \cdot (\alpha \cdot I_{t+1}^{sub} + H_{t+1}) - H_{t+1} \right] \\
&= \left(I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub} \right) \cdot (1 - \tau_D) - (1 - \tau_C) \cdot (1 - \tau_P) \cdot \left[I_{t+1}^{par} + (1 - y \cdot \tau_C) \cdot \alpha \cdot I_{t+1}^{sub} - y \cdot \tau_C \cdot H_{t+1} \right].
\end{aligned}$$

To show the impact of deductible interest expenses (H_{t+1}) and non-deductible interest expenses ($I_{t+1}^{sub,non}$), the tax shield, in case that $I_{t+1}^{par} = 0$ and $\alpha = 0$ is defined as

$$TS_{t+1}^{rec} = (1 - \tau_C) \cdot (1 - \tau_P) \cdot y \cdot \tau_C \cdot H_{t+1}.$$

A.3 Tax shield equations for the nor-case

If there is no reclassification of the dividends (nor-case) the after tax income of the parent is given by

$$\begin{aligned}
FCF_{t+1}^{L,nor,par} &= (1 - \tau_C) \cdot \left[EBIT_{t+1}^{par} - I_{t+1}^{par} + (1 - \alpha) \cdot I_{t+1}^{sub} \right] \\
&\quad + (1 - y \cdot \tau_C) \cdot \left[(1 - \tau_C) \cdot EBIT_{t+1}^{sub} - I_{t+1}^{sub} + \tau_C \cdot (\alpha \cdot I_{t+1}^{sub} + H_{t+1}) \right] \\
&= (1 - \tau_C) \cdot \left[EBIT_{t+1}^{par} - I_{t+1}^{par} + (1 - \alpha) \cdot I_{t+1}^{sub} \right] \\
&\quad + (1 - y \cdot \tau_C) \cdot \left[(1 - \tau_C) \cdot EBIT_{t+1}^{sub} - (1 - \alpha \cdot \tau_C) \cdot I_{t+1}^{sub} + \tau_C \cdot H_{t+1} \right] \\
&= (1 - \tau_C) \cdot \left(EBIT_{t+1}^{par} - I_{t+1}^{par} \right) + (1 - \tau_C) \cdot (1 - \alpha) \cdot I_{t+1}^{sub} \\
&\quad + (1 - y \cdot \tau_C) \cdot \left[(1 - \tau_C) \cdot EBIT_{t+1}^{sub} - (1 - \alpha \cdot \tau_C) \cdot I_{t+1}^{sub} + \tau_C \cdot H_{t+1} \right] \\
&= (1 - \tau_C) \cdot \left(EBIT_{t+1}^{par} - I_{t+1}^{par} \right) + (1 - \alpha) \cdot (1 - \tau_C) \cdot I_{t+1}^{sub} \\
&\quad + (1 - \tau_C) \cdot (1 - y \cdot \tau_C) \cdot EBIT_{t+1}^{sub} - (1 - \alpha \cdot \tau_C) \cdot (1 - y \cdot \tau_C) \cdot I_{t+1}^{sub} + (1 - y \cdot \tau_C) \cdot \tau_C \cdot H_{t+1} \\
&= (1 - \tau_C) \cdot \left[EBIT_{t+1}^{par} - I_{t+1}^{par} + (1 - y \cdot \tau_C) \cdot EBIT_{t+1}^{sub} \right] + (1 - \alpha) \cdot (1 - \tau_C) \cdot I_{t+1}^{sub} \\
&\quad - (1 - \alpha \cdot \tau_C) \cdot (1 - y \cdot \tau_C) \cdot I_{t+1}^{sub} + (1 - y \cdot \tau_C) \cdot \tau_C \cdot H_{t+1} \\
&= (1 - \tau_C) \cdot \left[EBIT_{t+1}^{par} - I_{t+1}^{par} + (1 - y \cdot \tau_C) \cdot EBIT_{t+1}^{sub} \right] \\
&\quad - \left[(1 - \alpha \cdot \tau_C) \cdot (1 - y \cdot \tau_C) - (1 - \alpha) \cdot (1 - \tau_C) \right] \cdot I_{t+1}^{sub} + (1 - y \cdot \tau_C) \cdot \tau_C \cdot H_{t+1}.
\end{aligned}$$

Hence the after-tax income of an individual investor of a levered firm is given by

$$\begin{aligned}
FCF_{t+1}^{L,nor} &= \left(I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub} \right) \cdot (1 - \tau_D) + (1 - \tau_P) \cdot \left[(1 - \tau_C) \cdot \left[EBIT_{t+1}^{par} - I_{t+1}^{par} + (1 - y \cdot \tau_C) \cdot EBIT_{t+1}^{sub} \right] \right. \\
&\quad \left. - \left[(1 - \alpha \cdot \tau_C) \cdot (1 - y \cdot \tau_C) - (1 - \alpha) \cdot (1 - \tau_C) \right] \cdot I_{t+1}^{sub} + (1 - y \cdot \tau_C) \cdot \tau_C \cdot H_{t+1} \right].
\end{aligned}$$

The tax shield under consideration of a thin-capitalization rule in the nor-case is calculated as

$$\begin{aligned}
TS_{t+1}^{nor} &= FCF_{t+1}^{L,nor} - FCF_{t+1}^U \\
&= (I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub}) \cdot (1 - \tau_D) + (1 - \tau_P) \cdot \left[(1 - \tau_C) \cdot [EBIT_{t+1}^{par} - I_{t+1}^{par} + (1 - y \cdot \tau_C) \cdot EBIT_{t+1}^{sub}] \right. \\
&\quad \left. - [(1 - \alpha \cdot \tau_C) \cdot (1 - y \cdot \tau_C) - (1 - \alpha) \cdot (1 - \tau_C)] \cdot I_{t+1}^{sub} + (1 - y \cdot \tau_C) \cdot \tau_C \cdot H_{t+1} \right] \\
&\quad - (1 - \tau_C) \cdot (1 - \tau_P) \cdot [EBIT_{t+1}^{par} + (1 - y \cdot \tau_C) \cdot EBIT_{t+1}^{sub}] \\
&= (1 - \tau_D) \cdot (I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub}) + (1 - \tau_P) \cdot \left[- (1 - \tau_C) \cdot I_{t+1}^{par} \right. \\
&\quad \left. - [(1 - \alpha \cdot \tau_C) \cdot (1 - y \cdot \tau_C) - (1 - \alpha) \cdot (1 - \tau_C)] \cdot I_{t+1}^{sub} + (1 - y \cdot \tau_C) \cdot \tau_C \cdot H_{t+1} \right] \\
&= (I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub}) \cdot (1 - \tau_D) - (1 - \tau_P) \cdot \left[(1 - \tau_C) \cdot I_{t+1}^{par} \right. \\
&\quad \left. + [(1 - \alpha \cdot \tau_C) \cdot (1 - y \cdot \tau_C) - (1 - \alpha) \cdot (1 - \tau_C)] \cdot I_{t+1}^{sub} - (1 - y \cdot \tau_C) \cdot \tau_C \cdot H_{t+1} \right].
\end{aligned}$$

To show the impact of deductible interest expenses (H_{t+1}) and non-deductible interest expenses ($I_t^{sub,non}$), the tax shield in case that $I_t^{par} = 0$ and $\alpha = 0$ is defined as

$$\begin{aligned}
TS_{t+1}^{nor} &= - (1 - \tau_P) \cdot \left[(1 - y) \cdot \tau_C \cdot (I_{t+1}^{sub,non} + H_{t+1}) - (1 - y \cdot \tau_C) \cdot \tau_C \cdot H_{t+1} \right] \\
&= - (1 - \tau_P) \cdot \left[(1 - y) \cdot \tau_C \cdot I_{t+1}^{sub,non} - (1 - \tau_C) \cdot y \cdot \tau_C \cdot H_{t+1} \right]
\end{aligned}$$

A.4 Demonstration for validity of $TS_{t+1}^N - TS_{t+1}^{TC} \leq 0$ when a thin-capitalization rule is not applicable

In case of a thin-capitalization rule the tax shield is defined in the rec- and nor-case as

$$TS_{t+1}^{rec} = (I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub}) \cdot (1 - \tau_D) - (1 - \tau_C) \cdot (1 - \tau_P) \cdot [I_{t+1}^{par} + (1 - y \cdot \tau_C) \cdot \alpha \cdot I_{t+1}^{sub} - y \cdot \tau_C \cdot H_{t+1}]$$

and

$$\begin{aligned}
TS_{t+1}^{nor} &= (I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub}) \cdot (1 - \tau_D) - (1 - \tau_P) \cdot \left[(1 - \tau_C) \cdot I_{t+1}^{par} \right. \\
&\quad \left. + [(1 - \alpha \cdot \tau_C) \cdot (1 - y \cdot \tau_C) - (1 - \alpha) \cdot (1 - \tau_C)] \cdot I_{t+1}^{sub} - (1 - y \cdot \tau_C) \cdot \tau_C \cdot H_{t+1} \right]
\end{aligned}$$

for

$$H_{t+1} = (1 - \alpha) \cdot r_D \cdot D_t^{sub} \cdot \frac{j}{h} \cdot \frac{E_t^{sub}}{D_t^{sub}} \cdot \frac{D^{TC}}{E^{TC}}.$$

The thin-capitalization applies, if

$$\frac{D^{TC}}{E^{TC}} \leq \frac{h \cdot D_t^{sub}}{j \cdot E_t^{sub}},$$

where $\frac{D^{TC}}{E^{TC}}$ is the fixed safe haven ratio, that has to be exceeded, $h \cdot D_t^{sub}$ and $j \cdot E_t^{sub}$ are the applicable debt and equity values for calculating the exceeding safe haven ratio.

The tax shield in case that a thin-capitalization rule applies is defined as

$$TS_{t+1} = TS_{t+1}^N - \max(TS_{t+1}^N - TS_{t+1}^{TC}, 0).$$

In the following, we want to demonstrate, the universality of

$$TS_{t+1}^N - TS_{t+1}^{TC} \leq 0$$

for the situation that a thin-capitalization rule does not apply. This is valid for

$$\frac{D^{TC}}{E^{TC}} \geq \frac{h \cdot D_t^{sub}}{j \cdot E_t^{sub}}.$$

First, we demonstrate for the rec-case:

$$\begin{aligned}
0 &\geq TS_{t+1}^N - TS_{t+1}^{rec} \\
0 &\geq (I_{t+1}^{par} - \alpha \cdot I_{t+1}^{sub}) \cdot (1 - \tau_D) - (1 - \tau_C) \cdot (1 - \tau_P) \cdot [I_{t+1}^{par} + (\alpha - y \cdot \tau_C) \cdot I_{t+1}^{sub}] \\
&\quad - \left[(I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub}) \cdot (1 - \tau_D) - (1 - \tau_C) \cdot (1 - \tau_P) \cdot [I_{t+1}^{par} - H_{t+1} + (1 - y \cdot \tau_C) \cdot (\alpha \cdot I_{t+1}^{sub} + H_{t+1})] \right] \\
0 &\geq (1 - \tau_C) \cdot (1 - \tau_P) \cdot \left[-I_{t+1}^{par} - (\alpha - y \cdot \tau_C) \cdot I_{t+1}^{sub} + I_{t+1}^{par} - H_{t+1} + (1 - y \cdot \tau_C) \cdot (\alpha \cdot I_{t+1}^{sub} + H_{t+1}) \right] \\
0 &\geq (1 - \tau_C) \cdot (1 - \tau_P) \cdot \left[-I_{t+1}^{par} - \alpha \cdot I_{t+1}^{sub} + y \cdot \tau_C \cdot I_{t+1}^{sub} + I_{t+1}^{par} - H_{t+1} + \alpha \cdot I_{t+1}^{sub} + H_{t+1} - y \cdot \tau_C \cdot \alpha \cdot I_{t+1}^{sub} - y \cdot \tau_C \cdot H_{t+1} \right] \\
0 &\geq (1 - \tau_C) \cdot (1 - \tau_P) \cdot [y \cdot \tau_C \cdot (1 - \alpha) \cdot I_{t+1}^{sub} - H_{t+1}] \\
0 &\geq (1 - \tau_C) \cdot (1 - \tau_P) \cdot y \cdot \tau_C \cdot [(1 - \alpha) \cdot I_{t+1}^{sub} - H_{t+1}] \\
0 &\geq (1 - \tau_C) \cdot (1 - \tau_P) \cdot y \cdot \tau_C \cdot \left[(1 - \alpha) \cdot r_D \cdot D_t^{sub} - (1 - \alpha) \cdot r_D \cdot D_t^{sub} \cdot \frac{j}{h} \cdot \frac{E_t^{sub}}{D_t^{sub}} \cdot \frac{D^{TC}}{E^{TC}} \right] \\
0 &\geq (1 - \tau_C) \cdot (1 - \tau_P) \cdot y \cdot \tau_C \cdot \left(D_t^{sub} - \frac{j}{h} \cdot E_t^{sub} \cdot \frac{D^{TC}}{E^{TC}} \right) \\
0 &\geq D_t^{sub} - \frac{j}{h} \cdot E_t^{sub} \cdot \frac{D^{TC}}{E^{TC}} \\
\frac{j}{h} \cdot E_t^{sub} \cdot \frac{D^{TC}}{E^{TC}} &\geq D_t^{sub} \\
\frac{D^{TC}}{E^{TC}} &\geq \frac{h \cdot D_t^{sub}}{j \cdot E_t^{sub}} \text{ (q.e.d.)}.
\end{aligned}$$

Now we demonstrate for the nor-case:

$$\begin{aligned}
0 &\geq TS_{t+1}^N - TS_{t+1}^{nor} \\
0 &\geq \left(I_{t+1}^{par} - \alpha \cdot I_{t+1}^{sub} \right) \cdot (1 - \tau_D) - (1 - \tau_C) \cdot (1 - \tau_P) \cdot \left[I_{t+1}^{par} + (\alpha - y \cdot \tau_C) \cdot I_{t+1}^{sub} \right] - \left[\left(I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub} \right) \cdot (1 - \tau_D) \right. \\
&\quad \left. - (1 - \tau_P) \cdot \left[(1 - \tau_C) \cdot I_{t+1}^{par} + [(1 - \alpha \cdot \tau_C) \cdot (1 - y \cdot \tau_C) - (1 - \alpha) \cdot (1 - \tau_C)] \cdot I_{t+1}^{sub} - (1 - y \cdot \tau_C) \cdot \tau_C \cdot H_{t+1} \right] \right] \\
0 &\geq (1 - \tau_P) \cdot \left[- (1 - \tau_C) \cdot I_{t+1}^{par} - (1 - \tau_C) \cdot (\alpha - y \cdot \tau_C) \cdot I_{t+1}^{sub} + (1 - \tau_C) \cdot I_{t+1}^{par} \right. \\
&\quad \left. + [(1 - \alpha \cdot \tau_C) \cdot (1 - y \cdot \tau_C) - (1 - \alpha) \cdot (1 - \tau_C)] \cdot I_{t+1}^{sub} - (1 - y \cdot \tau_C) \cdot \tau_C \cdot H_{t+1} \right] \\
0 &\geq (1 - \tau_P) \cdot \left[[(1 - \alpha \cdot \tau_C) \cdot (1 - y \cdot \tau_C) - (1 - \alpha) \cdot (1 - \tau_C) - (\alpha - y \cdot \tau_C) \cdot (1 - \tau_C)] \cdot I_{t+1}^{sub} - (1 - y \cdot \tau_C) \cdot \tau_C \cdot H_{t+1} \right] \\
0 &\geq (1 - \tau_P) \cdot \left[I_{t+1}^{sub} \cdot \tau_C \alpha \cdot I_{t+1}^{sub} - y \cdot \tau_C \cdot I_{t+1}^{sub} + y \cdot \tau_C \cdot \tau_C \cdot \alpha \cdot I_{t+1}^{sub} - I_{t+1}^{sub} + \alpha \cdot I_{t+1}^{sub} + \tau_C \cdot I_{t+1}^{sub} - \tau_C \cdot \alpha \cdot I_{t+1}^{sub} - \alpha \cdot I_{t+1}^{sub} \right. \\
&\quad \left. + y \cdot \tau_C \cdot I_{t+1}^{sub} + \tau_C \cdot \alpha \cdot I_{t+1}^{sub} - y \cdot \tau_C \cdot \tau_C \cdot I_{t+1}^{sub} - (1 - y \cdot \tau_C) \cdot \tau_C \cdot H_{t+1} \right] \\
0 &\geq (1 - \tau_P) \cdot \left[- \tau_C \cdot \alpha \cdot I_{t+1}^{sub} + y \cdot \tau_C \cdot \tau_C \cdot \alpha \cdot I_{t+1}^{sub} + \tau_C \cdot I_{t+1}^{sub} - y \cdot \tau_C \cdot \tau_C \cdot I_{t+1}^{sub} - (1 - y \cdot \tau_C) \cdot \tau_C \cdot H_{t+1} \right] \\
0 &\geq (1 - \tau_P) \cdot \tau_C \cdot \left[- \alpha \cdot I_{t+1}^{sub} + y \cdot \tau_C \cdot \alpha \cdot I_{t+1}^{sub} + I_{t+1}^{sub} - y \cdot \tau_C \cdot I_{t+1}^{sub} - (1 - y \cdot \tau_C) \cdot H_{t+1} \right] \\
0 &\geq (1 - \tau_P) \cdot \tau_C \cdot \left[(1 - \alpha) \cdot I_{t+1}^{sub} - y \cdot \tau_C \cdot (1 - \alpha) \cdot I_{t+1}^{sub} - (1 - y \cdot \tau_C) \cdot H_{t+1} \right] \\
0 &\geq (1 - \tau_P) \cdot \tau_C \cdot \left[(1 - y \cdot \tau_C) \cdot (1 - \alpha) \cdot I_{t+1}^{sub} - (1 - y \cdot \tau_C) \cdot H_{t+1} \right] \\
0 &\geq (1 - \tau_P) \cdot (1 - y \cdot \tau_C) \cdot \tau_C \cdot \left[(1 - \alpha) \cdot I_{t+1}^{sub} - H_{t+1} \right] \\
0 &\geq (1 - \tau_P) \cdot (1 - y \cdot \tau_C) \cdot \tau_C \cdot \left[(1 - \alpha) \cdot r_D \cdot D_t^{sub} - (1 - \alpha) \cdot r_D \cdot D_t^{sub} \cdot \frac{j}{h} \cdot \frac{E_t^{sub}}{D_t^{sub}} \cdot \frac{D^{TC}}{E^{TC}} \right] \\
0 &\geq (1 - \tau_P) \cdot (1 - y \cdot \tau_C) \cdot (1 - \alpha) \cdot \tau_C \cdot r_D \cdot \left[D_t^{sub} - D_t^{sub} \cdot \frac{j}{h} \cdot \frac{E_t^{sub}}{D_t^{sub}} \cdot \frac{D^{TC}}{E^{TC}} \right] \\
0 &\geq (1 - \tau_P) \cdot (1 - y \cdot \tau_C) \cdot (1 - \alpha) \cdot \tau_C \cdot r_D \cdot \left[D_t^{sub} - \frac{j}{h} \cdot E_t^{sub} \cdot \frac{D^{TC}}{E^{TC}} \right] \\
0 &\geq D_t^{sub} - \frac{j}{h} \cdot E_t^{sub} \cdot \frac{D^{TC}}{E^{TC}} \\
\frac{D^{TC}}{E^{TC}} \cdot \frac{j}{h} \cdot E_t^{sub} &\geq D_t^{sub} \\
\frac{D^{TC}}{E^{TC}} &\geq \frac{h \cdot D_t^{sub}}{j \cdot E_t^{sub}} \text{ (q.e.d.)}.
\end{aligned}$$

Appendix B Derivation of Tax Shield Equations and Explanations for Earnings Stripping Rules

In general, the after-tax income of an individual investor of an unlevered firm is given by

$$FCF_{t+1}^U = (1 - \tau_C) \cdot (1 - \tau_P) \cdot [EBIT_{t+1}^{par} + (1 - y \cdot \tau_C) \cdot EBIT_{t+1}^{sub}].$$

The subsidiary's tax shield including the deductible interest expense Ψ_{t+1}^{sub} is defined as

$$\begin{aligned} FCF_{t+1}^{L,ESR,sub} &= (1 - \tau_C) \cdot (EBIT_{t+1}^{sub} - \Psi_{t+1}^{sub}) - (I_{t+1}^{sub} - \Psi_{t+1}^{sub}) \\ &= EBIT_{t+1}^{sub} - \Psi_{t+1}^{sub} - \tau_C \cdot EBIT_{t+1}^{sub} + \tau_C \cdot \Psi_{t+1}^{sub} - I_{t+1}^{sub} + \Psi_{t+1}^{sub} \\ &= (1 - \tau_C) \cdot EBIT_{t+1}^{sub} - I_{t+1}^{sub} + \tau_C \cdot \Psi_{t+1}^{sub}. \end{aligned}$$

Hence, the after-tax income of the parent considering its own deductible interest income Ψ_{t+1}^{par} is given by

$$\begin{aligned} FCF_{t+1}^{L,ESR,par} &= (1 - \tau_C) \cdot [EBIT_{t+1}^{par} + (1 - \alpha) \cdot I_{t+1}^{sub} - \Psi_{t+1}^{par}] \\ &\quad + (1 - y \cdot \tau_C) \cdot [(1 - \tau_C) \cdot EBIT_{t+1}^{sub} - I_{t+1}^{sub} + \tau_C \cdot \Psi_{t+1}^{sub}] - (I_{t+1}^{par} - \Psi_{t+1}^{par}) \\ &= (1 - \tau_C) \cdot [EBIT_{t+1}^{par} + (1 - \alpha) \cdot I_{t+1}^{sub} - \Psi_{t+1}^{par}] \\ &\quad + (1 - y \cdot \tau_C) \cdot [(1 - \tau_C) \cdot EBIT_{t+1}^{sub} - I_{t+1}^{sub} + \tau_C \cdot \Psi_{t+1}^{sub}] - I_{t+1}^{par} + \tau_C \cdot \Psi_{t+1}^{par}. \end{aligned}$$

This implies for the after-tax income of an individual investor of a levered firm is given by

$$\begin{aligned} FCF_{t+1}^{L,ESR} &= (I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub}) \cdot (1 - \tau_D) + (1 - \tau_P) \cdot \left[(1 - \tau_C) \cdot [EBIT_{t+1}^{par} + (1 - \alpha) \cdot I_{t+1}^{sub} - \Psi_{t+1}^{par}] \right. \\ &\quad \left. + (1 - y \cdot \tau_C) [(1 - \tau_C) \cdot EBIT_{t+1}^{sub} - I_{t+1}^{sub} + \tau_C \cdot \Psi_{t+1}^{sub}] - I_{t+1}^{par} + \tau_C \cdot \Psi_{t+1}^{par} \right]. \end{aligned}$$

The tax shield under consideration of an earnings stripping rule is calculated as

$$\begin{aligned}
TS_{t+1}^{ESR} &= FCF_{t+1}^{L,ESR} - FCF_{t+1}^U \\
&= (I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub}) \cdot (1 - \tau_D) + (1 - \tau_P) \cdot \left[(1 - \tau_C) \cdot [EBIT_{t+1}^{par} + (1 - \alpha) \cdot I_{t+1}^{sub} - \Psi_{t+1}^{par}] \right. \\
&\quad \left. + (1 - y \cdot \tau_C) \cdot [(1 - \tau_C) \cdot EBIT_{t+1}^{sub} - I_{t+1}^{sub} + \tau_C \cdot \Psi_{t+1}^{sub}] - I_{t+1}^{par} + \tau_C \cdot \Psi_{t+1}^{par} \right] \\
&\quad - (1 - \tau_C) \cdot (1 - \tau_P) \cdot [EBIT_{t+1}^{par} + (1 - y \cdot \tau_C) \cdot EBIT_{t+1}^{sub}] \\
&= (I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub}) \cdot (1 - \tau_D) + (1 - \tau_P) \cdot \left[(1 - \tau_C) \cdot (1 - \alpha) \cdot I_{t+1}^{sub} \right. \\
&\quad \left. - (1 - y \cdot \tau_C) \cdot I_{t+1}^{sub} + (1 - y \cdot \tau_C) \cdot \tau_C \cdot \Psi_{t+1}^{sub} - I_{t+1}^{par} + \tau_C \cdot \Psi_{t+1}^{par} \right] \\
&= (I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub}) \cdot (1 - \tau_D) + (1 - \tau_P) \cdot \left[[(1 - \alpha) \cdot (1 - \tau_C) - (1 - y \cdot \tau_C)] \cdot I_{t+1}^{sub} \right. \\
&\quad \left. + (1 - y \cdot \tau_C) \cdot \tau_C \cdot \Psi_{t+1}^{sub} + \tau_C \cdot \Psi_{t+1}^{par} - I_{t+1}^{par} \right] \\
&= (I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub}) \cdot (1 - \tau_D) - (1 - \tau_P) \cdot \left[I_{t+1}^{par} - [(1 - \alpha) \cdot (1 - \tau_C) - (1 - y \cdot \tau_C)] \cdot I_{t+1}^{sub} \right. \\
&\quad \left. - (1 - y \cdot \tau_C) \cdot \tau_C \cdot \Psi_{t+1}^{sub} - \tau_C \cdot \Psi_{t+1}^{par} \right] \\
&= (I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub}) \cdot (1 - \tau_D) - (1 - \tau_P) \cdot \left[I_{t+1}^{par} + [(1 - y \cdot \tau_C) - (1 - \alpha) \cdot (1 - \tau_C)] \cdot I_{t+1}^{sub} \right. \\
&\quad \left. - \tau_C \cdot \Psi_{t+1}^{par} - (1 - y \cdot \tau_C) \cdot \tau_C \cdot \Psi_{t+1}^{sub} \right].
\end{aligned}$$

To show the influence of an earnings stripping rule, we define the difference between the tax shield without any interest limitation TS_{t+1}^N with the tax shield under consideration of an earnings stripping rule

TS_{t+1}^{ESR} as

$$\begin{aligned}
TS_{t+1}^N - TS_{t+1}^{ESR} &= \left(I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub} \right) \cdot (1 - \tau_D) - (1 - \tau_C) \cdot (1 - \tau_P) \cdot \left[I_{t+1}^{par} + (\alpha - y \cdot \tau_C) \cdot I_{t+1}^{sub} \right] \\
&\quad - \left[\left(I_{t+1}^{par} + \alpha \cdot I_{t+1}^{sub} \right) \cdot (1 - \tau_D) - (1 - \tau_P) \cdot \left[I_{t+1}^{par} + [(1 - y \cdot \tau_C) - (1 - \alpha) \cdot (1 - \tau_C)] \cdot I_{t+1}^{sub} \right] \right. \\
&\quad \left. - (1 - y \cdot \tau_C) \cdot \tau_C \cdot \Psi_{t+1}^{sub} - \tau_C \cdot \Psi_{t+1}^{par} \right] \\
&= (1 - \tau_P) \cdot \left[- (1 - \tau_C) \cdot I_{t+1}^{par} - (1 - \tau_C) \cdot (\alpha - y \cdot \tau_C) \cdot I_{t+1}^{sub} \right. \\
&\quad \left. + I_{t+1}^{par} + [(1 - y \cdot \tau_C) - (1 - \alpha) \cdot (1 - \tau_C)] \cdot I_{t+1}^{sub} - \tau_C \cdot \Psi_{t+1}^{par} - (1 - y \cdot \tau_C) \cdot \tau_C \cdot \Psi_{t+1}^{sub} \right] \\
&= (1 - \tau_P) \cdot \left[- I_{t+1}^{par} + \tau_C \cdot I_{t+1}^{par} - [(1 - \tau_C) \cdot (\alpha - y \cdot \tau_C) - [(1 - y \cdot \tau_C) - (1 - \alpha) \cdot (1 - \tau_C)]] \cdot I_{t+1}^{sub} \right. \\
&\quad \left. + I_{t+1}^{par} - \tau_C \cdot \Psi_{t+1}^{par} - (1 - y \cdot \tau_C) \cdot \tau_C \cdot \Psi_{t+1}^{sub} \right] \\
&= (1 - \tau_P) \cdot \left[\tau_C \cdot I_{t+1}^{par} - (\alpha - \tau_C \cdot \alpha - y \cdot \tau_C + y \cdot \tau_C^2 - 1 + y \cdot \tau_C + 1 - \alpha - \tau_C + \tau_C \cdot \alpha) \cdot I_{t+1}^{sub} \right. \\
&\quad \left. - \tau_C \cdot \Psi_{t+1}^{par} - (1 - y \cdot \tau_C) \cdot \tau_C \cdot \Psi_{t+1}^{sub} \right] \\
&= (1 - \tau_P) \cdot \left[\tau_C \cdot I_{t+1}^{par} - (y \cdot \tau_C^2 - \tau_C) \cdot I_{t+1}^{sub} - \tau_C \cdot \Psi_{t+1}^{par} - (1 - y \cdot \tau_C) \cdot \tau_C \cdot \Psi_{t+1}^{sub} \right] \\
&= (1 - \tau_P) \cdot \left[\tau_C \cdot I_{t+1}^{par} - (y \cdot \tau_C - 1) \cdot \tau_C \cdot I_{t+1}^{sub} - \tau_C \cdot \Psi_{t+1}^{par} - (1 - y \cdot \tau_C) \cdot \tau_C \cdot \Psi_{t+1}^{sub} \right] \\
&= (1 - \tau_P) \cdot \tau_C \cdot \left(I_{t+1}^{par} - (y \cdot \tau_C - 1) \cdot I_{t+1}^{sub} - \Psi_{t+1}^{par} - (1 - y \cdot \tau_C) \cdot \Psi_{t+1}^{sub} \right) \\
&= (1 - \tau_P) \cdot \tau_C \cdot \left[I_{t+1}^{par} + (1 - y \cdot \tau_C) \cdot I_{t+1}^{sub} - \Psi_{t+1}^{par} - (1 - y \cdot \tau_C) \cdot \Psi_{t+1}^{sub} \right] \\
&= (1 - \tau_P) \cdot \tau_C \cdot \left[I_{t+1}^{par} - \Psi_{t+1}^{par} + (1 - y \cdot \tau_C) \cdot (I_{t+1}^{sub} - \Psi_{t+1}^{sub}) \right].
\end{aligned}$$

In comparison to a thin-capitalization rule, an earnings stripping rule can also have a positive influence on the tax shield as non-deductible interest expenses can be carried forward and deduct the taxable income in future periods.

References

- Arzac, E. R. and Glosten, L. R., 'A Reconsideration of Tax Shield Valuation', *European Financial Management*, Vol. 11(4), 2005, pp. 453-461.
- Buettner, T. Overesch, M., Schreiber, U. and Wamser, G., 'The impact of thin-capitalization rules on the capital structure of multinational firms. A reconsideration of tax shield valuation', *Journal of Public Economics*, Vol. 96(11-12), 2012, pp. 930-938.
- Cooper, I. A. and Nyborg, K. G., 'The value of tax shields IS equal to the present value of tax shields', *Journal of Financial Economics*, Vol. 81(1), 2006, pp. 215-225.
- Cooper, I. A. and Nyborg, K. G., 'Tax-Adjusted Discount Rates with Investor Taxes and Risky Debt', *Financial Management*, Vol. 37(2), 2008, pp. 365-379.
- Couch, R., Dothan, M. and Wu, W., 'Interest Tax Shields: A Barrier Options Approach', *Review of Quantitative Finance and Accounting*, Vol. 39(1), 2012, pp. 123-146.
- Dempsey, M., 'Consistent Cash Flow Valuation with Tax-Deductible Debt: a Clarification', *European Financial Management*, Vol. 19(4), 2013, pp. 830-836.
- Eberl, S., 'Weitere Erkenntnisse zum Steuervorteil von Fremdkapital nach der Unternehmensteuerreform 2008', *Zeitschrift fuer betriebswirtschaftliche Forschung*, Vol. 61(3), 2009, pp. 251-282.
- Fernandez, P., 'The value of tax shields is NOT equal to the present value of tax shields', *Journal of Financial Economics*, Vol. 73(1), 2004, pp. 145-165.
- Fieten, P., Kruschwitz, L., Laitenberger, J., Loeffler, A., Tham, J., Vélez-Pareja, I. and Wonder, N., 'Comment on "The value of tax shields is NOT equal to the present value of tax shields"', *The Quarterly Review of Economics and Finance*, Vol. 45(1), 2005, pp. 184-187.
- Graham, J. R., 'How Big Are the Tax Benefits of Debt?', *Journal of Finance*, Vol. 55(5), 2000, pp.1901-1941.
- Graham, J. R., 'Taxes and Corporate Finance: A Review', *Review of Financial Studies*, Vol. 16(4), 2003, pp. 1075-1129.
- Knauer, A., Lahmann, A., Pfluecke, M. and Schwetzler, B., 'How Much Do Private Equity Funds Benefit From Debt-related Tax Shields?', *Journal of Applied Corporate Finance*, Vol. 26(1), 2014, pp. 85-93.
- Knauer, T. and Sommer, F., 'Interest barrier rules as a response to highly leveraged transactions: Evidence from the 2008 German business tax reform', *Review of Accounting and Finance*, Vol. 11(2), 2012, pp. 206-232.
- Koziol, C., 'A simple correction of the WACC discount rate for default risk and bankruptcy costs', *Review of Quantitative Finance and Accounting*, Vol. 42, 2013, pp. 653-666.
- Massari, M., Roncaglio, F. and Zanetti, L., 'On the Equivalence between the APV and the wacc Approach in a Growing Leveraged Firm', *European Financial Management*, Vol. 14(1), 2008, pp. 152-162.
- Miles, J. A. and Ezzell, J. R., 'The Weighted Average Cost of Capital, Perfect Capital Markets, and Project Life: A Clarification', *Journal of Financial and Quantitative Analysis*, Vol. 15(3), 1980, pp. 719-730.

- Miles, J. A. and Ezzell, J. R., 'Reformulating Tax Shield Valuation: A Note', *Journal of Finance*, Vol. 40(5), 1985, pp. 1485-1492.
- Miller, M. H., 'Debt and Taxes', *Journal of Finance*, Vol. 32(2), 1977, pp. 261-275.
- Modigliani, F. and Miller, M. H., 'The Cost of Capital, Corporation Finance, and the Theory of Investment', *The American Economic Review*, Vol. 48(3), 1958, pp. 261-297.
- Modigliani, F. and Miller, M. H., 'Corporate Income Taxes and the Cost of Capital: A Correction', *The American Economic Review*, Vol. 53(3), 1963, pp. 433-443.
- Molnár, P. and Nyborg, K. G., 'Tax-adjusted Discount Rates: a General Formula under Constant Leverage Ratios', *European Financial Management*, Vol. 19(3), 2013, pp. 419-428.
- Myers, S. C., 'Interactions of Corporate Financing and Investment Decisions - Implications for Capital Budgeting', *Journal of Finance*, Vol. 29(1), 1974, pp. 1-25.
- Overesch, M. and Wamser, G., 'Corporate tax planning and thin-capitalization rules: evidence from a quasi-experiment', *Applied Economics*, Vol. 42(5), 2010, pp. 563-573.
- Rasmussen, P. N., 'Direct EU regulation for Private Equity and Hedge funds The real economy comes first', Commission conference on private equity and hedge funds, February 2009.
- van Binsbergen, J. H., Graham, J. R. and Yang, J., 'The Cost of Debt', *Journal of Finance*, Vol. 65(6), 2010, pp. 2089-2136.
- Weichenrieder, A. J. and Windschbauer, H., 'Thin-capitalization rules and company responses experience from German legislation', *CESifo working paper*, 2008, p. 2456.